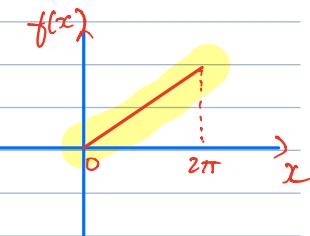


Fourier Series



Example 1 : Find the Fourier series representing

$$f(x) = x, \quad 0 < x < 2\pi$$

and sketch its graph from $x = -4\pi$ to $x = 4\pi$

Solution :

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right]$$

assumes that the f^n is defined over 0 to $2L$

The corresponding Fourier coefficients are

STEP ONE $a_0 = \frac{1}{L} \int_{-2L}^{2L} f(x) dx$

f^n is defined from 0 to 2π
 $2L = 2\pi$
 $\Rightarrow L = \pi$

STEP TWO $a_n = \frac{1}{L} \int_{-2L}^{2L} f(x) \cos \frac{n\pi x}{L} dx$

STEP THREE $b_n = \frac{1}{L} \int_{-2L}^{2L} f(x) \sin \frac{n\pi x}{L} dx$

and integrations are over a single interval in x of $2L$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos nx + b_n \sin nx \right]$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx = \frac{1}{\pi} \int_0^{2\pi} x dx = \frac{1}{\pi} \left[\frac{x^2}{2} \right]_0^{2\pi} = 2\pi$$

$$a_0 = 2\pi$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_0^{2\pi} x \cos nx dx = \frac{1}{\pi} \left\{ x \frac{\sin nx}{n} \Big|_0^{2\pi} - \int_0^{2\pi} \frac{1}{n} \sin nx dx \right\}$$

$$\int_a^b u v' dx = uv \Big|_a^b - \int_a^b u' v dx$$

$$\Rightarrow a_n = \frac{1}{\pi} \left[\frac{x \sin nx}{n} + \frac{\cos nx}{n^2} \right]_0^{2\pi}$$

$$\Rightarrow a_n = \frac{1}{\pi} \left[\frac{2\pi \sin 2n\pi}{n} + \frac{\cos 2n\pi}{n^2} - 0 - \frac{\cos 0}{n^2} \right]$$

$$= \frac{1}{\pi} \left[0 + \frac{1}{n^2} - 0 - \frac{1}{n^2} \right]$$

$$\Rightarrow a_n = 0$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx dx = \frac{1}{\pi} \int_0^{2\pi} \underbrace{x}_u \underbrace{\sin nx}_v dx$$

$$\int_a^b u v' dx = uv \Big|_a^b - \int_a^b u' v dx$$

$$\Rightarrow b_n = \frac{1}{\pi} \left\{ -x \frac{\cos nx}{n} \Big|_0^{2\pi} - \int_0^{2\pi} 1 \cdot \left(-\frac{\cos nx}{n} \right) dx \right\}$$

$$= \frac{1}{\pi} \left[-x \frac{\cos nx}{n} + \frac{\sin nx}{n^2} \right]_0^{2\pi}$$

$$= \frac{1}{\pi} \left[-2\pi \frac{\cos 2n\pi}{n} + \frac{\sin 2n\pi}{n^2} + 0 - 0 \right]$$

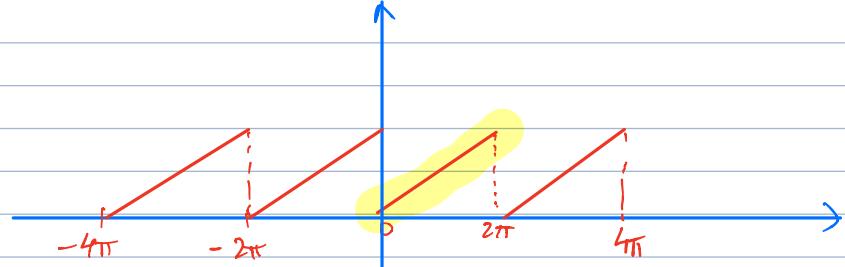
$$= \frac{1}{\pi} \left[-\frac{2\pi}{n} + 0 + 0 - 0 \right]$$

$$\Rightarrow b_n = -\frac{2}{n}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

$$\Rightarrow f(x) = \pi + \sum_{n=1}^{\infty} [0 - \frac{2}{n} \sin nx]$$

$$\Rightarrow \boxed{f(x) = \pi - 2 \left[\sin x + \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x + \dots \right]}$$



$$a_0 = \pi$$

$$f(x) = -f(x)$$

odd f^n

sine is an odd

