

Fourier Series

$$f(x) = f(x+L) = f(x+2L) = \dots$$

where L is the period

period: $2L$

Intro

$\cos(nx)$ $\sin(nx)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right]$$

} general period $2L$

The corresponding Fourier coefficients are

STEP ONE

$$a_0 = \frac{1}{2L} \int f(x) dx$$

STEP TWO

$$a_n = \frac{1}{2L} \int f(x) \cos \frac{n\pi x}{L} dx$$

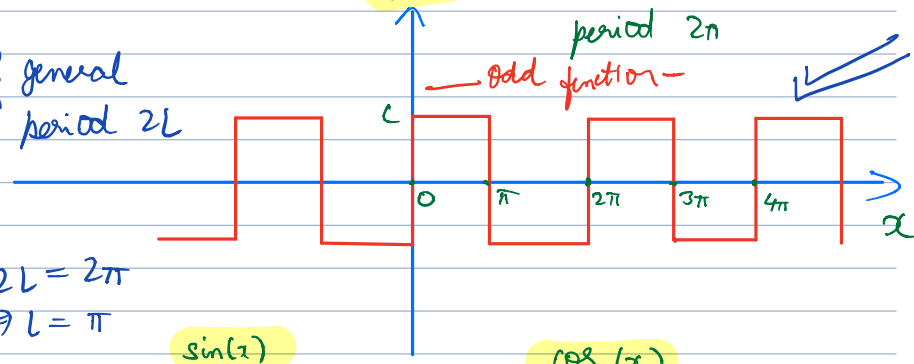
STEP THREE

$$b_n = \frac{1}{2L} \int f(x) \sin \frac{n\pi x}{L} dx$$

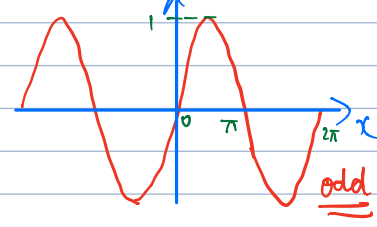
and integrations are over a single interval in x of $2L$

$$2L = 2\pi$$
$$\Rightarrow L = \pi$$

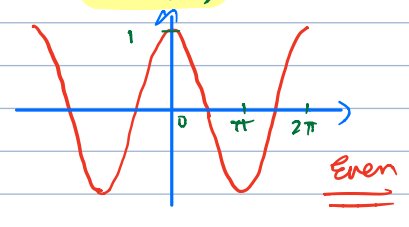
$f(x)$



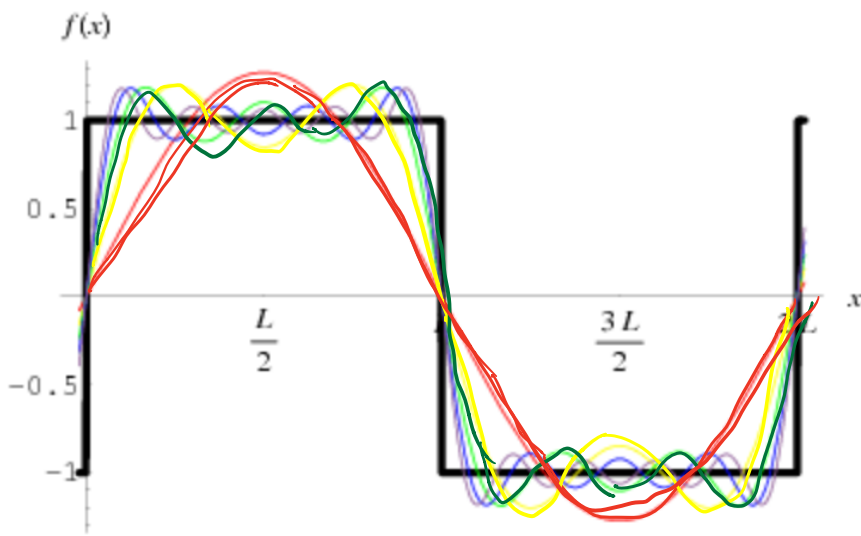
$\sin(x)$



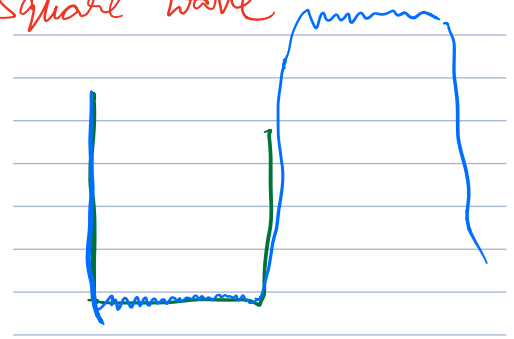
$\cos(x)$



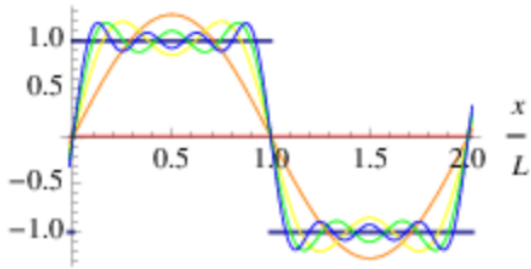
$$f(x) = \frac{a_0}{2} + a_1 \cos(x) + a_2 \cos(2x) + a_3 \cos(3x) + \dots$$
$$+ b_1 \sin(x) + b_2 \sin(2x) + b_3 \sin(3x) + \dots$$



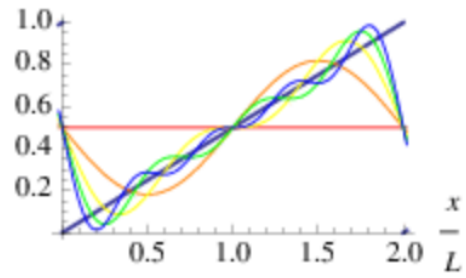
→ Square wave



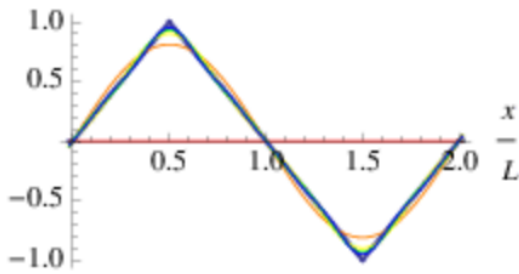
square wave



sawtooth wave



triangle wave



semicircle

