Department of Physics and Astrophysics
University of Delhi

M.Sc. (Physics) - Semester II Examination - May 2017
PHYS-407 Statistical Mechanics

PART B

Instructions for PART B

Answer any three questions out of four on the given answer sheet. Each question carries 17 marks. This part of the question paper need not be returned with your answer sheets. Values of a few constants are given at the end.

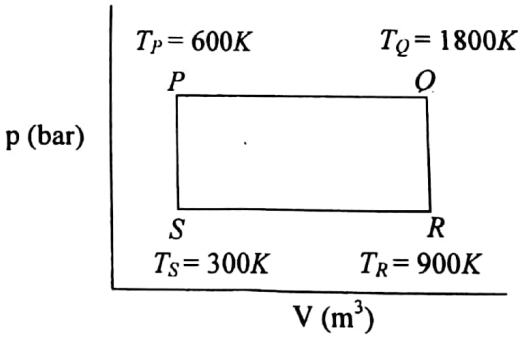
1. (i) 3.5 moles of an ideal gas is expanded from 450K and an initial pressure of 5 bar to a final pressure of 1 bar, and $C_{P,m} = 5R/2$.

   (a) If the expansion is quasi static and isothermal then calculate the work done. In this case does the heat flow into or out of the bath? Calculate the change in entropy. Calculate the change in entropy of the bath. (5)

   (b) If the gas expands in one stage expansion to final pressure of 1 bar while still being in contact with the heat bath, calculate the work done and change in entropy of the bath. (4)

   (c) In which of the two cases is the change in entropy of the gas greater? Justify your answer. (2)

   (ii)



   The gas is subjected to a reversible closed cycle $P \rightarrow Q \rightarrow R \rightarrow S \rightarrow P$ as shown in the above diagram. Temperatures at the points P, Q, R and S are indicated. Calculate the value of $\int \frac{dQ}{T}$ along each part of the cycle. Also find out the value of $\oint \frac{dQ}{T}$ and explain its significance. (6)

2. Consider a system of two indistinguishable particles. These particles are distributed in energy states $E_1 = 0, E_2 = \varepsilon, E_3 = E_4 = 2\varepsilon$.

   (a) List all the microstates of the system with their energies separately for bosons and fermions. (4)

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(b) Calculate the partition function of the system for both bosons and fermions. (4)

(c) Calculate the average energy in the two cases at $T \rightarrow 0$K and $T \rightarrow \infty$. (5)

(d) Calculate the entropy of the system in the two cases at a finite temperature T. (4)

3. (i) The Hamiltonian for N massless relativistic free particles in 3 space dimensions is given by $H = \sum_{i=1}^{N} |p_i| c$. If the total constant energy of the system is E [$= \sum_{i=1}^{N} \varepsilon_i$] ,

(a) Find out the radius R of the constant energy sphere in $3N$-dimensional momentum space as a function of E. (2)

(b) If V is the volume of the system, find out the total number of accessible microstates and the expression for entropy.

[Hint: The volume of a sphere of radius R in $3N$ dimension is given by $\frac{\pi^{3N/2}}{\frac{3N}{2}!} R^{3N}$] (4)

(c) Find the relation between the temperature T and the total energy E. Also find out the expression for pressure and hence the equation of state for the system. (5)

(ii) Consider a stellar gas of Hydrogen (H) atoms in thermal equilibrium at temperature T such that the average kinetic energy per H atom ($\frac{3}{2} k_B T$) is 1 eV. The energy levels of hydrogen atoms are $\varepsilon_n = -\alpha/n^2$ eV, where α is a dimensionless constant and $n = 1, 2, 3$ etc. The degeneracy of the n-th level is $2n^2$. Show that the ratio of the number of atoms in the second excited state (n =3) to the number in the ground state (n =1) is $9e^{-4\alpha/3}$. Assume that the number of H atoms is fixed in the stellar gas. (6)

4. (i) Consider a 1D periodic Ising lattice of 3 spins S_1, S_2, S_3 in an external magnetic field h (here $S_i = \pm 1$). The neighboring spins are mutually interacting with strength J.

(a) Write down the Hamiltonian of this system. (2)

(b) Write down all the microstates of this system and their energies. In the case $h = 0$ how many micro states are there with energy $E = 0$ and $E = J$? (6)

(ii) Explain the mean field approximation. Show the existence of paramagnetic - ferromagnetic phase transition by invoking the mean field approximation in a 2D lattice of N spins. (9)

Some physical constants:

Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K Universal gas constant $R = 8.31$ J/ mol. K

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