This question paper contains one sheet with both sides printed Your Roll No.....

M.Sc.(PHYSICS), IV-Semester 2018 May, University of Delhi

Paper-PHYS-576: General Theory of Relativity & Cosmology-II

Time: 3 hours Maximum Marks: 70

(Write your Roll No. on the top of this question paper immediately on receipt)

Answer **all five** questions in Section-A and **any three** questions in Section-B.
All notations have their usual meaning.

Section-A

1(a) Model the star as a perfect fluid with an interior metric given by the line-element:

$$ds^2 = -e^{\alpha(r)}dt^2 + e^{\beta(r)}dr^2 + r^2 d\Omega^2 \ .$$

Obtain expression(s) for all the non-zero components of $T_{\mu\nu}$. Draw the conformal diagram for the Schwarzschild black hole and label it.

(b) State Birkoff's theorem with an example in GTR.

(5+3)-marks

2(a) Define a maximally symmetric n-dimensional space and specify all the isometries therein. Derive an expression for the Riemann tensor in a maximally symmetric 3-space.

(b) Analyze the line-element $ds^2 = -dt^2 + R^2(t)[d\rho^2 + \sin^2\rho\, d\Omega^2]$ under an appropriate coordinate transformation to obtain a special case of the Robertson-Walker metric and comment on this special case.

(5+3)-marks

3(a) Define the characteristics of a perfect fluid and write down the energy-momentum-stress (EMS) tensor $T^{\mu\nu}$ for it. Write down the $T^{\mu\nu}$ for a radiation dominated universe and derive a relation between the energy density and normal stress. Does the relation signify any classical symmetry?

(b) State Hubble's law and express it as a mathematical equation. What is the estimated value of the Hubble constant for a spatially flat universe with $\rho = 10^{-29}$ gm/cc?

(5+3)-marks

4(a) Write down the linearized gravitational wave equation in free space. State the auxiliary conditions and mathematically show a reason behind them. Use an appropriate gauge and obtain the gravitational plane wave solution.

(b) Use diagram(s) to illustrate the plus (+) and cross (×) polarizations of a gravitational wave independently. State an empirical formula and find its spin.

(5+3)-marks

5(a) Use the geodesic equation for a FRW universe.
Show that : $\dfrac{dt}{d\lambda} = \dfrac{\omega_0}{a}$, where λ = affine parameter, ω_0 = constant, a = scale factor.

(b) A photon is emitted with an energy E and is measured by a comoving observer with velocity U^μ. Use $dt/d\lambda$ in **Q5(a)** to derive the red shift Z.

(5+3)-marks

Section-B

6(a) Show that the FRW universe is described by the fluid equation: $\dot{\rho} + 3H[\rho + p] = 0$.

(b) Use $p = \omega\rho$ for a constant ω and derive the evolution of ρ with scale factor.

(5+5)-marks

7(a) Use the Friedmann equation for an empty universe and show that a positively curved geometry is forbidden while a negatively curved geometry is allowed.

(b) A line-element is given by $ds^2 = -dt^2 + t^{2q}\left(dr^2 + r^2 d\Omega^2\right)$. Why is it called cosmological? What value of q describes the Milne universe? What is the characteristic of this universe?

(5+5)-marks

8(a) The non-zero affine connections on S^2 are: $\Gamma^{\theta}_{\phi\phi} = -(\sin\theta\cos\theta)$ and $\Gamma^{\phi}_{\theta\phi} = \cot\theta$. Obtain all the Killing equations in explicit form.

(b) Explain how Eddington-Finkelstein coordinates resolve the issue with the light cone at the event horizon of a Schwarzschild black hole in static coordinates? Draw appropriate diagram(s).

(5+5)-marks

9(a) Draw a labelled Kruskal diagram with time-like curves only for the extended Schwarzschild geometry. What does every point in the diagram represent? State two merits of Kruskal coordinates over static coordinates.

(b) Draw geometries of five distinct space-like slices in Kruskal diagram. What is an Einstein-Rosen bridge? What does it connect?

(5+5)-marks

10(a) What are Chandrasekhar and Oppenheimer-Volkoff limits? Explain them with examples. Compare the variation of radial coordinate with mass respectively for white dwarfs, neutron star and black hole in one diagram.

(b) State the polytropic equation of state and explain where it is used.

Physically interpret the negative norm: $U_\mu U^\mu = -1$, where U^μ = observer's 4-velocity.

(6+4)-marks