Astronomy & Astrophysics-II (PHYS-575)

Internal Assessment (March 8, 2018)

(Answer any two questions.)

Time : 1 hrs 30 min *Maximum Marks* : 30

1. For a spherically symmetric star (with total mass M_0, radius R, density $\rho(r)$ and pressure $P(r)$) in hydrostatic equilibrium, where r is the radial distance from the centre of the star,

(a) show that,

$$\int_0^R \frac{GM(r)}{r^3} dM(r) = 4\pi \int_0^R P(r) dr \ ,$$

where $M(r)$ is the mass enclosed within the radius r. (6)

(b) Prove that,

$$P_c \leq P(r) + \frac{1}{2} G \left(\frac{4\pi}{3}\right)^{1/3} \rho_c^{4/3} M^{2/3}(r)$$

given that,

$$P_c > P(r) + \frac{GM^2(r)}{8\pi r^4} > \frac{GM_0^2}{8\pi R^4} \ ,$$

where $P_c \equiv P(0)$ and $\rho_c \equiv \rho(0)$ are the central pressure and central density, respectively.

(9)

2. Consider a highly conducting magnetosphere near the surface of an aligned rotator spinning with an angular velocity $\vec{\omega} = \omega \hat{k}$ and having a magnetic dipole moment $\vec{\mu} = \frac{1}{2} B_p R^3 \ \hat{k}$. Assuming that a electric charge density ρ_e and a macroscopic current density $J_e = \rho_e \vec{v} = \rho_e (\vec{r} \times \vec{\omega})$ develops near the surface of the rotating neutron star, show that,

$$\rho_e = - \frac{1}{2\pi c} \ \vec{\omega}.\vec{B} \left(1 - \frac{v^2}{c^2}\right)^{-1} .$$

(15)

3. (a) Assuming the Robertson-Walker line element $ds^2 = c^2 dt^2 - a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$ for $k = 0$, show that the corresponding luminosity distance is given by,

$$D_L(z) = \frac{2c}{H_0}[1 + z - \sqrt{1+z}]$$

given that $a(t) = A\, t^{2/3}$, $D_L(z) = a(t_0)\,(1+z)\, r(z)$ and $H_0 = \frac{2}{3\, t_0}$.

(10)

(b) A spinning star of initial radius R_i, period of rotation $P_i = 3 \times 10^8$ s and surface magnetic field $B_i = 10^3$ G collapses to form a neutron star with surface magnetic field $B_f = 10^{13}$ G. Find the final period of rotation of the neutron star. (5)