# Astronomy & Astrophysics-II (PHYS-575)

## M.Sc. 4-th semester examination (May 19, 2018)

(Answer any five questions. Note that Gaussian units have been used for Maxwell electrodynamics.)

*Time* : 3 hrs

*Maximum Marks* : 70

**1 (a)** A hydrogen-rich star of mass $M$ and radius $R$ is given to be in virial equilibrium.

(i) If the average temperature of the star is $\bar{T}$, how is it related to $M$ and $R$? (3)

(ii) Estimate $\bar{T}$ for the Sun using the result of 1(a). (1)

(iii) If this star is connected to another far away, virialized, hydrogen-rich star of same mass $M$ but radius $> R$ with a long, hypothetical thermal conductor of negligible mass, can the stars ever reach thermal equilibrium? Why? How will both the stars evolve? (3)

**(b)** (i) If $B_\nu(T)$ represents the specific intensity of black body radiation at temperature $T$ show that when radiation of frequency $\nu$ passes through matter with absorption coefficient $\alpha_\nu$, the change in the specific intensity $I_\nu$ as it moves an infinitesimal distance $ds$ is given by,

$$\frac{dI_\nu}{ds} = \alpha_\nu[B_\nu(T(s)) - I_\nu(s)] \ .$$

(Assume that the matter is locally in thermal equilibrium) (5)

(ii) Obtain $B_\nu(T)$ as a function of $\nu$, $T$ and fundamental constants, given that the specific energy density of black body radiation is,

$$u_\nu(T) = \frac{8\pi h\nu^3}{c^3[\exp{(h\nu/k_BT)} - 1]}$$

(2)

**2 (a)** The iron core, weighing 2.5 $M_\odot$, of a massive star undergoes gravitational free fall from an initial radius of $2 \times 10^4$ km to a final radius 10.5 km during which protons and electrons of the core combine to form neutrons and neutrinos. If 99 % of the liberated gravitational energy is taken away by neutrinos, estimate the total energy carried away by the neutrinos. (4)

**(b)** (i) Assume that the large scale geometry of the universe is described by the Robertson-Walker line element and the total flux $F$ from a cosmic source lying at redshift $z$ is given by $F = L/4\pi D_L^2(z)$, where $L$ and $D_L$ are the total luminosity and luminosity-distance, respectively, of the source. Derive a relation between the luminosity distance, redshift and the radial coordinate of the source. (7)

(ii) In the Steady State Cosmology (SSC), the redshift-radial coordinate relation is given by,

$$z = \frac{H_0}{c}a(t_0)r \ .$$

If a galaxy at redshift $z = 3$ is observed to have a total flux $F = 6 \times 10^{-23}$ erg $\text{cm}^{-2}\text{s}^{-1}$, determine the total luminosity of the galaxy according to the SSC. (3)

**3** **(a)** If a pulsar of radius $R$ and polar cap magnetic field $B_p$ is taken to be an aligned rotator with an exterior magnetic field given by,

$$\vec{B}_{out}(\vec{r}) = \frac{B_p R^3}{r^3}[\cos\theta\ \hat{e}_r + \frac{1}{2}\sin\theta\ \hat{e}_\theta] ,$$

determine the tangential component of the electric field just outside the pulsar surface assuming zero surface current. (5)

**(b)** A cosmic source of mass $M$ and luminosity $L$ is powered by the differential rotation and viscosity of an accretion disc around it, with inner and outer radii of the disc being $r_I$ and $r_O$, respectively. If the accretion rate $\dot{M} \equiv \frac{dM}{dt}$ is steady then:

(i) determine the angular momentum per unit mass of the disc at a radius $r$, and estimate the rate at which the spin angular momentum of the source is changing, (3)

(ii) and find an upper limit on $\dot{M}$ arising due to the radiation pressure. (6)

**4** **(a)** A pulsar has a spin period of 47 milli-seconds. Find the distance from the spin axis of the pulsar beyond which the charge particles present in the magnetosphere **cannot** co-rotate with the pulsar. Does this distance depend on the angle between the spin axis and magnetic dipole moment of the pulsar? (3)

**(b)** The power radiated by an accelerating particle with charge $q$ is given by,

$$P_{em} = \frac{2q^2}{3c^3}\gamma^4[a_\perp^2 + \gamma^2 a_\parallel^2]$$

where $\gamma \equiv (\sqrt{1 - v^2/c^2})^{-1}$, $a_\perp$ and $a_\parallel$ are the Lorentz factor, acceleration normal to the velocity and acceleration along the velocity, respectively. Show that the power radiated by an electron moving along a curved magnetic field line outside the light-cylinder of a pulsar is,

$$P_{em} = \frac{2e^2}{3c^3}\frac{\gamma^4\ v^4}{R_{curv}^2}$$

where $v$ and $R_{curv}$ are the electron speed and the radius of curvature of the magnetic field lines, respectively.

(7)

**(c)** Using the Heisenberg uncertainty principle for an electron localized within a size $\sim n^{-1/3}$, where $n$ is the average number density of electrons in a white dwarf, estimate the maximum mass for a stable, non-rotating white dwarf. Why is the proton's momentum not included in the calculation? (4)

**OR**

Starting from the definition of the Hubble parameter, $H(t) \equiv \frac{1}{a(t)}\frac{da}{dt}$, show that the nearby cosmic sources (i.e. $z < 0.1$) approximately satisfy the Hubble's law,

$$z = \frac{H_0}{c}d ,$$

where $d$ is a measure of the source distance and $H_0 \equiv H(t_0)$, $t_0$ being the present age of the universe.

(4)

5 **(a)** A polytrope has pressure-density relation given by $P \propto \rho^2$. Solve the Lane-Emden equation exactly to obtain $\rho(r)$. What is the mass-radius relation in this case? (5)

**(b)** If solar magnetic fields, $B \approx 10^3$ G, are generated in the outer convection zone of depth $d \approx 2 \times 10^{10}$ cm, where the electron density is $n \approx 10^{23}$ cm$^{-3}$, estimate the relative drift speed of electrons with respect to the protons. (3)

**(c)** Consider charge particles moving relativistically in a distant radio-source, gyrating in circles with a speed $v$ around a uniform magnetic field $\vec{B} = B\ \hat{e}_z$. If an observer's line of sight is perpendicular to the z-axis, estimate the observed band-width of the synchrotron radiation spectrum. (Assume that the radiation from the charge particles are beamed in a narrow cone with angle $\gamma^{-1}$, $\gamma$ being the Lorentz factor.) (6)

6 **(a)** Find a formal solution to the radiative transfer equation,

$$\frac{dI_\nu}{ds} = \alpha_\nu[B_\nu(T(s)) - I_\nu(s)]$$

and prove that $I_\nu \to B_\nu$, the black body intensity, when the optical depth $\tau_\nu \to \infty$. (6)

**(b)** Show that for a thin accretion disc around an astrophysical compact object of mass $M$,

$$\rho\, T\, \frac{ds_{en}}{dt} = \frac{9}{4}\eta \frac{GM}{r^3}$$

where $\eta$, $s_{en}$, $\rho$ and $T$ are the viscosity coefficient, entropy density, gas density and temperature, respectively, of the accretion disc.

(8)

**Useful data:**

(a) $M_\odot = 2 \times 10^{33}$ gm, $R_\odot = 6.96 \times 10^{10}$ cm.

(b) Lane-Emden equation:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\ \theta^n$$

(c) Robertson-Walker line-element:

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

(d) $H_0 = 70$ km/s/Mpc.

(e) Hydrostatic equilibrium condition for a spherical star:

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

(f) Thomson scattering cross-section: $\sigma_T = 0.665 \times 10^{-24}$ cm$^2$