University of Delhi
Department of Physics and Astrophysics
M.Sc. Physics (Semester IV)
PHY580 : Advanced Numerical Techniques
Final Examination (2018)

Time: 1.5 hours *Maximum Marks: 40*

Some useful equations are given on the backside of this question paper.

Instructions:

- **Please write down your name, roll number and college name on the question paper as well as the answer book.**
- **Use your home area to make the program. (Do not make any subdirectory.)**
- **Please name your program and other files using the following convention: finalant-2018-FirstName.c. Use the same convention to name any other file.**
- **It is compulsory to write the entire C program in the answer book.**

Question: The dataset $\{x_i, y_i\}$, where $i = 1, 2, ...7$, given below is to be fitted using a Cubic spline, $y(x) = Ay_j + By_{j+1} + Cy_j'' + Dy_{j+1}''$ where A, B, C, and D have their usual meanings . Use Natural boundary conditions; $y_1'' = 0$ and $y_7'' = 0$.

x_i	0	6	10	13	17	20	28
y_i	6.67	17.33	42.67	37.33	30.10	29.31	28.74

1. Obtain the tridiagonal set in the Matrix form $Py'' = Q$ that will provide y_j'', where $j = 2, 3...6$. **Print** the matrices P (dimension: 5x5) and Q (dimension: 5x1).
2. Solve the tridioagnal set using Gauss Elimination method to obtain y_j''. **Print** the matrix y'' (dimension: 5x1).
3. Using A, B, C, D and y_j'', obtain the **value** of the Cubic spline $y(x)$ at $x = 12$.
4. **Plot** the Cubic spline function $y(x)$ from $x = 0$ to $x = 28$ in steps of $x = 1$.

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This paper contains only one page.

All questions are compulsory.

1. (a) For the three data-points, $x_0 = 3$, $x_1 = 8$, and $x_2 = 15$, find the second Lagrange interpolating polynomial for the function $f(x) = \sqrt{1+x}$.
   (b) Use this interpolating polynomial to estimate the value of the function $f(x)$ at $x = 9$.

   (6+2=8)

2. (a) The dataset $\{x_i, y_i, \sigma\}$ has to be fitted using the function $f(x) = Ax - Bx^3$, where A and B are the two unknown parameters, and $i = 1, 2...N$. Here, σ is the constant error on each value of y.
      i. Set-up the χ^2 function for this curve-fitting problem.
      ii. Using the χ^2 minimization technique, derive the minimization equations.
      iii. Solve the minimization equations to estimate the best-fit values of the parameters A and B.

   (2+3+5=10)

   (b) How would the values of the best fit parameters A and B change, if each value of y_i has a different error σ_i. Answer briefly (derivations are not required). (2)

3. Let x be a random variable distributed according to the following probability distribution function:

$$f(x) = Nx^{\beta-1}\exp(-\frac{x^\beta}{\alpha})$$

where α and β are two free parameters, and $x \geq 0$; $\alpha, \beta > 0$.

   (a) Find the normalization constant N of the probability distribution function.
   (b) Let u be a random variable distributed according to the uniform distribution in the interval $[0, 1]$. Using the inverse transformation method, express x in terms of u.

   (3+7=10)