[This question paper contains 2 printed pages]

Your Roll No. ....

UNIVERSITY OF DELHI

Department of Physics & Astrophysics

M.Sc. Physics, I Semester, November-December 2016

Physics 402, (Quantum Mechanics-I)

Maximum Marks: 70

Time: 3 Hrs

(Write your Roll No. on the top immediately on receipt of this question paper)

Answer any FIVE questions

1. (a) Check whether the following functions are linearly independent or linearly dependent on the real x−axis: $f_1(x) = 8$, $f_2(x) = x^3$, $f_3(x) = e^{2x}$. [2].

   (b) Obtain the condition for which the operator $|\psi\rangle\langle\psi|$ would become a projection operator. [1].

   (c) Consider a system whose state is given in terms of an orthonormal set of three vectors $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\phi_3\rangle$ as: $|\alpha\rangle = \frac{\sqrt{3}}{2}|\phi_1\rangle + \frac{2}{3}|\phi_2\rangle + \frac{\sqrt{2}}{3}|\phi_3\rangle$ Verify if $|\alpha\rangle$ is normalized. Also calculate the probability of finding the system in any one of the states $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\phi_3\rangle$. [2].

   (d) Investigate if the vector states defined by: $|\alpha\rangle = 3i|\phi_1\rangle - 7i|\phi_2\rangle$ and $|\beta\rangle = -|\phi_1\rangle + 2i|\phi_2\rangle$ where $|\phi_1\rangle$ and $|\phi_2\rangle$ are orthonormal, satisfy the Schwarz inequality. [2].

   (e) Prove that the transformation matrix relating one orthonormal basis to another basis is unitary. [3].

   (f) The operators S_x and S_y are defined as follows: $S_x := \frac{1}{2}\hbar(|+\rangle\langle-| + |-\rangle\langle+|)$; $S_y := \frac{1}{2}\hbar(-i|+\rangle\langle-| + i|-\rangle\langle+|)$. Obtain the matrix representations for the operators S_x and S_y. [4].

2. (a) Using the notions of the coordinate and momentum basis, construct the transformation function $\langle x|p\rangle$ and also establish the relations connecting the coordinate-space and momentum-space wave-functions and vice-versa. [3+3].

   (b) Using the definition of an infinitesimal space translation operator, prove that two successive space translations in two mutually perpendicular directions x and y commute. [2].

   (c) Define an anti-linear and anti-unitary operator and prove that the inner product of two state kets considered under the reversal of motion (time-reversal) is preserved. [4].

   (d) Calculate the direct (or the outer) product $|\alpha\rangle\langle\beta|$ where $\langle\alpha| := (+3i, 2-i, 4)$ and $\langle\beta| := (2, +i, 2+3i)$. [2].

3. (a) Show that three arbitrary Hermitian operators A, B and C which satisfy the commutation relation $[A, B] = iC$ in the Schrodinger representation would satisfy the same commutation relation in the Heisenberg and Dirac (or interaction) representations also. [4]

   (b) Prove that the expectation value of an operator A remains the same in the Heisenberg and Schrodinger representations. [2]

(c) Construct the base kets for the Heisenberg representation and obtain the appropriate (wrong-sign) Schrodinger equation obeyed by the Heisenberg representation base kets. [3 + 3].

(d) State the transformation properties obeyed by polar and pseudo-vectors and by the true and pseudo-scalars under the parity transformation. [2].

4. (a) The Hamiltonian operator of a one-dimensional harmonic oscillator is defined by: $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$ where m, and ω are some constants. The two non-Hermitian operators a and $a^\dagger$ are also given:

$$a := \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{ip}{m\omega}\right) \quad , \quad a^\dagger := \sqrt{\frac{m\omega}{2\hbar}}\left(x - \frac{ip}{m\omega}\right)$$

Calculate the matrix elements: $\langle n'|\, x\, |n\rangle$ and $\langle n'|\, p\, |n\rangle$ for the above harmonic oscillator where $|n\rangle$ is an eigenstate of the Number operator $N := a^\dagger a$ with eigenvalue n. [6]

(b) Given $a\,|0\rangle = 0$ construct the ground state wave function $\langle x|\,0\rangle \equiv \psi_0(x)$ for the above oscillator. [4]

(c) *Investigate in detail whether the* Number operator $N := a^\dagger a$ for the above oscillator would remain *invariant or not in the* Heisenberg and in Schrodinger representations. [4]

5. (a) Prove the commutation relation $[\hat{T}_2, \hat{T}_1] = i\hbar\hat{T}_3$ where, $\hat{T}_1 = \hat{p}^2 - \hat{x}^2$, $\hat{T}_2 = \hat{x}\hat{p} + \hat{p}\hat{x}$ and $\hat{T}_3 = \hat{p}^2 + \hat{x}^2$. [4.5]

(b) Obtain the expression of angular momentum operator, $\hat{L}_y$ in terms of spherical polar coordinates. [4.5]

(c) Construct the matix elements $\langle j'm'|\, \hat{j}_\pm\, |jm\rangle$ for $j = 1$, where $\hat{j}_\pm$ are ladder operators. [5]

6. (a) Derive the following recursion relation for Clebsch-Gordan coefficients

$$\begin{aligned}\sqrt{(j \pm m)(j \mp m + 1)}\,\langle j_1, j_2; m_1, m_2|\, j, m\rangle &= \sqrt{(j_1 \pm m_1)(j_1 \mp m_1 + 1)}\,\langle j_1, j_2; m_1 \mp 1, m_2|\, j, m \mp 1\rangle \\ &+ \sqrt{(j_2 \pm m_2)(j_2 \mp m_2 + 1)}\,\langle j_1, j_2; m_1, m_2 \mp 1|\, j, m \mp 1\rangle\end{aligned}$$

[4]

(b) Consider $j_1 = j_2 = 1$. Calculate the Clebsch-Gordan cofficients $\langle j_1, j_2; m_1, m_2|\, j, m\rangle$ for $j = 2$. [10]