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Your Roll No.: 12104

UNIVERSITY OF DELHI

Department of Physics & Astrophysics

M.Sc. (Physics), I Semester, 2012

Phys-403 (Electromagnetic Theory & Electrodynamics)

Maximum Marks: 70

Time: 3 hrs.

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer *any five* questions

All questions carry equal marks

1. (a) Consider two frames S and S' related by the transformation:

$$x_\mu \longrightarrow x'_\mu = a_\mu{}^\nu\, x_\nu$$

obtain the values of all the matrix elements of the matrix: $a_\mu{}^\nu$ for Lorentz transformations when the frames S and S' have their axes parallel and S' is are moving relative to S with velocity v along the x direction.

4 marks

(b) Given the *covariant* components of the electromagnetic field tensor $F_{\mu\nu}$, that transform as covariant components of a tensor of rank -2 under the Lorentz transformation described in part (a) of this question, obtain the transformation equations for the components of the electric and magnetic fields in the frame S' in terms of the corresponding components in the frame S.

10 marks

2. (a) Consider the motion of a particle having a charge q in a uniform electromagnetic field configuration having **E** and **B** parallel to each other. Set up and solve the equations of motion and describe the trajectory of the particle.

8 marks

(b) Given a frame of reference S having uniform electric and magnetic fields **E** and **B** perpendicular to each other, and having their magnitudes satisfying: $|\mathbf{E}| < |\mathbf{B}|$ {note: in SI units this inequality reads $|\mathbf{E}| < c|\mathbf{B}|$}, show that there exists a frame S' in which the electric field identically vanishes. Discuss the trajectory of a particle introduced in S at rest at $t = 0$.

6 marks

3. (a) Show that the Lorentz force equation for a relativistic particle of charge q in an electromagnetic field can be written as

$$K^\mu \equiv \frac{dP^\mu}{d\tau} = \frac{q}{c} F^{\mu\alpha} U_\alpha$$

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in Gaussian units [the right hand side being $qF^{\mu\alpha}U_\alpha$ in S.I. units], with K^μ, P^μ and U^α being the contravariant components of the four force, four momentum and four velocity of the particle respectively.

7 marks

(b) Starting from the covariant form of Maxwell's equations, show that in the Lorentz gauge the electromagnetic field satisfies

$$\Box A^\mu = \mu_o J^\mu$$

where $\mu_o = \frac{4\pi}{c}$ in Gaussian units.

3 marks

(c) Show that the homogeneous Maxwell's equations follow from the vanishing divergence of the dual electromagnetic field tensor, i.e.:

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

4 marks

4. Show that the electromagnetic retarded 4-potential defined (in Gaussian units) by:

$$A^\mu(\mathbf{x}, t) \equiv \frac{1}{c}\int \frac{\mathbf{j}^\mu(\mathbf{x}', t')d^3\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|},$$

with the retarded time defined by:

$$t' = \left[t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right]$$

is a solution of the equation

$$\Box A^\mu = \partial^\alpha \partial_\alpha A^\mu = \frac{4\pi}{c}\mathbf{j}^\mu$$

where the symbols have their usual meanings.

14 marks

radiation

5. Starting from the expressions for the fields $\mathbf{E}$ and $\mathbf{B}$ for a ~~point charge~~ in arbitrary motion, derive the Larmor's formula for the total power radiated by a non-relativistic charged particle. Further, obtain the expressions for the power radiated per unit solid angle for the relativistic motion ($\mathbf{v} \to \mathbf{c}$) of a charged accelerated particle when the velocity $\mathbf{v}$ is parallel to the acceleration of the particle, a, and discuss the pattern of angular distribution of the emitted radiation for this case.

14 marks

6. (a) Consider the Lagrangian density of a relativistic point particle defined by

$$\mathcal{L}_1 \equiv \frac{1}{2}\left[\eta^{-1}\left(\dot{x}^\mu\right)^2 - m^2c^2\eta\right]$$

with $x^\mu \equiv x^\mu(\tau)$, $\eta \equiv \eta(\tau)$ and $\dot{x}^\mu = \frac{dx^\mu}{d\tau}$, and obtain the Euler Lagrange equations for motion under variations of $\eta(\tau)$ and $x^\mu(\tau)$. Show that the above Lagrangian density is classically equivalent to the Lagrangian density defined by

$$\mathcal{L}_2 \equiv -mc\sqrt{-\left(\dot{x}^\mu\right)^2}$$

where the symbols have their usual meanings.

6 marks

(b) Consider the action of a relativistic point particle defined by

$$S \equiv -mc \int \sqrt{-g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}\, d\tau$$

where the symbols have their usual meanings.

i. Calculate the canonical Hamiltonian density of the theory.

4 marks

ii. Show that the action of the theory is reparametrization invariant.

4 marks

7. (a) Consider an electromagnetic field configuration in which all quantities (sources and fields) vary harmonically in time.

(i) Establish the Poynting theorem for a cycle average (in the steady state).

(ii) Show that the cycle average of the total power emitted by a bounded source distribution is independent of the surface enclosing the source.

9 marks

(b) Prove that:

$$2\epsilon^{\alpha\beta\gamma\delta}\partial_\gamma A_\delta \, \epsilon_{\alpha\beta\mu\nu}\partial^\mu A^\nu = E^2 - B^2.$$

5 marks

8. (a) Show that in the radiation zone, the electromagnetic field of an arbitrary oscillating source is given by:

$$\vec{\mathrm{B}} = ik\hat{n} \times \vec{A}; \qquad \vec{\mathrm{E}} = \vec{\mathrm{B}} \times \hat{n}; \qquad |\vec{\mathrm{E}}| = |\vec{\mathrm{B}}|$$

5 marks

(b) Consider an electromagnetic source distribution with vanishing electric dipole as well as vanishing electric quadrupole moments.

(i) Express the electromagnetic field in the radiation zone in terms of the magnetic dipole moment of the source distribution.

(ii) Determine the total power emitted from an arbitrary surface that completely encloses the source, in any zone.

9 marks