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UNIVERSITY OF DELHI

Department of Physics & Astrophysics
M.Sc. (Physics), I Semester, December 3, 2011
Physics-403 (Eleectromagnetic Theory & Electrodynamics)

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Time: 3 hrs.

Maximum Marks: 70

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer *any five* questions. All questions carry equal marks.

1. (a) Show that the two inhomogeneous Maxwell's equations that contain sources (charge and current densities) constitute the covariant equation

$$\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu$$

5 marks

(b) Show that $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is a solution to

$$\partial_\mu \, {}^\star F^{\mu\nu} = 0 \,, \quad \text{where} \quad {}^\star F^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

3 marks

(c) A photon of energy 10 KeV collides with another photon of energy $\mathcal{E}$. Find the minimum value of $\mathcal{E}$ so that the collision produces a e^+e^- pair.

6 marks

2. (a) Consider an electromagnetic field configuration in which all quantities (sources and fields) vary harmonically in time.

(i) Establish the Poynting theorem for a cycle average (in the steady state).

(ii) Show that the cycle average of the total power emitted by a bounded source distribution is independent of the surface enclosing the source.

8 marks

(b) A particle of charge q and rest mass m_0 is released with zero initial velocity in a region of space containing a uniform electric field $\vec{\mathbf{E}}$ and a uniform magnetic field $\vec{\mathbf{B}}$ in the y and z directions respectively.

(i) Find the condition necessary for the existence of a Lorentz frame in which the magnetic field vanishes.

(ii) Solve the Lorentz force equation to describe the trajectory of the particle.
for the above field configuration (in part 1)

6 marks

3. (a) Consider a cylindrically symmetric oscillating source distribution with vanishing electric dipole and vanishing magnetic dipole strengths.

   (i) Express the electriomagnetic field in the radiation zone in terms of the quadrupole moment of the source charge distribution.

   (ii) Determine the total power emitted from a rectangular box that completely encloses the source, in the 'near' zone.

9 marks

(b) Show that the electromagnetic field tensor satisfies the equation

$$\Box F_{\mu\nu} = \frac{4\pi}{c}\left(\partial_\mu J_\nu - \partial_\nu J_\mu\right) ,$$

where $\Box \equiv \partial_\mu \partial^\mu$ is the D'Alembertian operator (also denoted by $\Box^2$ in some text).

5 marks

4. (a) For a particle of charge q moving with instantaneous velocity $\vec{v}$, the non-relativistic expression for the total power radiated is given by the Larmor result

$$\mathcal{P} = \frac{2}{3}\frac{q^2 \dot{v}^2}{c^3} ,$$

where $\dot{v} \equiv dv/dt$ is the particle's acceleration.

Generalize this expression to the case of a relativistically moving particle, to show that the total power is given by

$$\mathcal{P} = \frac{2}{3}\frac{q^2}{c}\gamma^6\left[\dot{\vec{\beta}}^2 - \left(\vec{\beta}\times\dot{\vec{\beta}}\right)^2\right] ,$$

where $\vec{\beta} = \vec{v}/c$ and $\gamma = (1-\beta^2)^{-1/2}$.

8 marks

(b) Two particles, both of rest mass m_o and moving with velocities $\vec{v}$ and $-\vec{v}$, collide and form a particle of mass M in an inertial frame S.

   (i) Make a Lorentz transformation to a frame S' in which the second particle is at rest. Express the speed of the final particle in S' in terms of the rest masses of the particles.

   (ii) Determine the four momentum vectors of all the three particles in this frame.

6 marks

5. (a) Show that the components of electric and magnetic fields due to a particle of charge q, moving with constant velocity $\vec{v}$ in the x-direction, are given by

$$E_x = \frac{q\gamma}{s^3}(x - vt) \; ; \quad E_y = \frac{q\gamma}{s^3}y \; ; \quad E_z = \frac{q\gamma}{s^3}z \; , \quad \text{and}$$

$$B_x = 0 \; ; \quad B_y = -\frac{q\beta\gamma}{s^3}z \; ; \quad B_z = \frac{q\beta\gamma}{s^3}y \; ,$$

where $\beta = v/c$, $\gamma = (1-\beta^2)^{-1/2}$, and

$$s = \left[\gamma^2(x-vt)^2 + y^2 + z^2\right]^{1/2} .$$

7 marks

(b) In a 20 GeV electron synchrotron, electrons are accelerated in a circular orbit of radius 200 m. Assuming very slow variation of the electron's momentum in magnitude, calculate in the extreme relativistic conditions, the energy loss per cycle due to emission of radiation.
[Given: Electron's rest energy $\mathcal{E}_0 = m_0 c^2 = 0.5$ MeV, and
Classical electron radius: $r_0 = e^2/\mathcal{E}_0 = 2.8 \times 10^{-15}$ m.]

7 marks

6. (a) Explain the *gauge invariance* of the electromagnetic field tensor $F^{\mu\nu}$, and write down the Lorentz and Coulomb gauge conditions. Are the gauge functions uniquely fixed in Lorentz and Coulomb gauges? If yes, why? If no, determine the residual gauge transformations in which the chosen (Lorentz or Coulomb) gauge is retained.

5 marks

(b) Given the expression for the power emitted per unit solid angle by a linearly accelerated particle (of charge q and instantaneous speed $v = c\beta$)

$$\frac{dP}{d\Omega} = \frac{q^2}{4\pi c} \frac{\left|\hat{n} \times \left(\hat{n} \times \dot{\vec{\beta}}\right)\right|^2}{(1 - \hat{n} \cdot \vec{\beta})^5},$$

where $\hat{n}$ is unit vector directed towards the observer from the emitter, at the emission time.

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(i) Determine the angle ϑ between $\hat{n}$ and $\vec{\beta}$ for which the power is maximum.

(ii) Show that for small ϑ, and in the extreme relativistic limit ($\beta \to 1$), the above expression reduces to

$$\frac{dP}{d\Omega} = \frac{8\gamma^8}{\pi} \frac{q^2 \dot{\beta}^2}{c} \frac{(\gamma\vartheta)^2}{[1 + (\gamma\vartheta)^2]^5}.$$

9 marks

7. (a) Starting from the covariant retarded Green's function solution to the wave operator

$$D(\underline{x} - \underline{x}') = \frac{1}{2\pi} \Theta(t - t') \, \delta\left(\|\underline{x} - \underline{x}'\|^2\right),$$

and taking the four current corresponding to a particle of charge q with four velocity $U^\mu(\tau) = dr^\mu/d\tau$

$$J^\mu = qc \int d\tau \, U^\mu(\tau) \, \delta^4\left(\underline{x} - \underline{r}(\tau)\right),$$

derive the expression for the vector potential in the Lorentz gauge

$$A^\mu(\underline{x}) = \left[\frac{q \, U^\mu(\tau)}{U_\lambda(\tau) \, (x^\lambda - r^\lambda(\tau))}\right]_{ret}$$

Here τ is the charge's proper time, and $\underline{r}(\tau)$ is the trajectory of the charged particle in space-time. [The underbar notation in the arguments, say $\underline{x}$, means a four vector with components x^λ.]

9 marks

(b) Consider a charged particle in relativistic motion. Show that the angular distribution of the power emitted is related to the angular distribution of Power detected by

$$\frac{dP}{d\Omega}(t_{em}) = \frac{dP}{d\Omega}(t_{det}) \left[1 - \hat{n} \cdot \vec{\beta}\right],$$

where the terms have their usual meaning.

5 marks