25/11/2014

Your Roll No. . . . .

Department of Physics and Astrophysics
University of Delhi

M.Sc. / I Sem.

PHY 401 : CLASSICAL MECHANICS

Three hours | Maximum Marks : 70

(Answer any five questions.)

(a) The Lagrangian for a Born-Infeld-like theory is given by $L = V(x)\sqrt{1-\dot{x}^2}$. Obtain the equation of motion and the Hamiltonian. (5)

(b) A point mass m is attached to two fixed vertical poles a distance 2ℓ apart by means of two massless springs, each of natural length ℓ and spring constant k. The mass is constrained to move in the vertical plane defined by the two poles.

i. Construct the Lagrangian for the general motion of the system. (3)

ii. Identify the equilibrium configuration. (2)

iii. For the special case of small disturbances, obtain the equations of motion and solve them. (4)

(a) A bead of mass μ can move smoothly along a fixed vertical pole placed at a point P on the ground. A particle of mass M can slide smoothly on a fixed horizontal straight line passing through P. The bead and the particle are connected by a rigid massless rod.

i. Construct the Lagrangian for the system and find the equations of motion as the bead slides down the vertical pole. (2.5)

ii. If the rod started from a nearly vertical position and with vanishing velocity, solve the equations of motion for the bead and the particle for the regime when the angle subtended by the rod with the vertical is still small. (2.5)

(b) A hard ball is bouncing vertically from a hard surface. The height to which the ball bounces is $h_{\max}$. Assume that the impact with the ground is elastic.

i. What is the Hamiltonian of the system? Use the Hamiltonian equations of motion to plot the phase space trajectories. (3)

ii. Evaluate the abridged action (the action variable). (2)

iii. Suppose the material of the ball evaporates isotropically with time so that the mass decreases at a rate dm/dt. As a result $h_{\max}$ also changes with time at a rate $dh_{\max}/dt$. Find the relation between dh_{max}/dt and dm/dt when dm/dt is much smaller than m/T, where T is the time taken for the ball to fall from h_{max} to the ground. (4)

3. (a) A fixed force centre scatters a particle of mass m according to the law $\vec{F}(\vec{r}) = k\,\vec{r}/r^4$, where k is a constant. The particle approaches from infinity with an initial velocity v_{in} and an impact parameter ρ.

i. Calculate the distance of closest approach $r_{\min}$ for both positive and negative values of k. (2)

ii. Sketch the particle trajectory, indicating the impact parameter and angular deflection (χ) for positive and negative k. (2)

iii. For positive k, calculate $d\chi/d\rho$. (3)

iv. For positive k, determine the differential scattering cross section. (3)

(b) Consider an isosceles triangle ABC such that $AB = AC = 2\ell = 2 \times BC$. Two identical masses ($m$ each) are placed at the vertices B and C, while a mass $2m$ is placed at the vertex A. If this system is considered to be a rigid body, calculate its principal moments of inertia. (4)

4. (a) Consider the phase space transformation $(q, p) \to (Q, P)$ given by

$$Q = q^a \exp(b\,p + \gamma\, t^2) \qquad P = p^a \exp(-b\,p + \eta\, t^2)$$

where a, b, γ, η are constants.

   i. What are the most general values of a, b, γ, η for which this is a canonical transformation? (2.5)

   ii. Obtain the corresponding generating function. (2.5)

(b) Consider a particle of mass m restricted to move in one dimension subject to a time-dependent force $F(t)$ which is independent of the phase space coordinates (x, p).

   i. Determine the Lagrangian and the Hamiltonian for this system. (2)

   ii. Obtain the Hamilton-Jacobi equation. (2)

   iii. Solve the Hamilton-Jacobi equation using the following Ansatz for Hamilton's principal function

$$S(x, \vec{\alpha}, t) = g_1(t, \vec{\alpha})\, x + g_2(t, \vec{\alpha}),$$

   where, $\vec{\alpha}$ is a constant vector. Express $g_{1,2}(t, \vec{\alpha})$ in terms of $F(t)$. Given the initial conditions $x(0) = x_0$ and $p(0) = p_0$, determine $\vec{\alpha}$. (3)

   iv. If $F(t) = F_0 \cos(\omega t)$, compute the integrals. (2)

5. (a) The equations of motion for a system with one degree of freedom are given by

$$\dot{q} = q\,p^2\,, \qquad \dot{p} = q\,p$$

Is this a Hamiltonian system? If yes, what is corresponding Hamiltonian? If not, why not? (4)

(b) Consider a particle of mass m moving in one dimension and subject to a periodic potential,

$$V(x) = V(x + 2na),$$

where n is an integer. In the interval $-a \le x \le a$, the potential is given by $V(x) = |V_0\, x/a|$ with V_0 being a positive constant.

   i. For energy $E < V_0$, show that the motion is periodic and calculate the time period. (3)

   ii. For $E > V_0$, calculate the action (J) and angle (θ) variable in terms of x and E. (3)

   iii. For $E \gg V_0$, calculate E in terms of J and θ. (2)

   iv. Sketch the phase space orbit for the three cases $E > V_0$, $E = V_0$ and $E < V_0$. (2)

6. (a) A thin uniform rectangular sheet of mass m and sides A and B (with $A > B$) rotates with constant angular speed about one of the two diagonals. Calculate the kinetic energy and angular momentum in an inertial frame. If the axis of rotation is fixed in this frame, what is the torque acting on the sheet? Ignore gravity. (8)

(b) A central force potential in 3-dimensions is given by, $V(r) = \alpha r$ where $\alpha > 0$.

   i. Calculate the radius/radii of stable circular orbit(s). (1)

   ii. Are there unbounded trajectories? If not, give reasons. If yes, give the conditions for their existence. (2)

   iii. Consider two particles in stable circular orbits of radii R_1 and R_2. What is the ratio of the two time periods of revolution? (3)