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3251 **A**

M. Sc. / I Sem.

PHYSICS 2010

Paper – PHYS-401 : Classical Mechanics

(Admission of 2009 and onwards)

Time : 3 hours Maximum Marks :70

(Write your Roll No. on the top of immediately on receipt of this question paper).
Attempt any **five** questions.

1. (a) Explain the D'Alembert's principle and derive the Euler Lagrange equations of motion for a system involving conservative and non-conservative forces. [7]

   (b) Derive the differential equation for the orbit of a particle moving under a central inverse square law of force using Hamilton's equations of motion of the problem. [4]

   (c) Prove that the isotropy of space leads to the conservation of angular momentum. [3]

2. (a) Explain the Hamilton's principle of least action and derive the Hamilton's canonical equations of motion for a system with n degrees of freedom using the δ - variation technique . [7]

   (b) Set up the Lagrangian for the motion of a charged particle moving in an electromagnetic field and obtain the Euler Lagrange equations of motion of the problem. [7]

3. (a) A particle of mass m moves in a 3-dimensional Euclidean space in a force field whose potential in spherical coordinates is given by (with $c(\phi)$ = constant :)

$$V(r,\theta,\phi) := \left[a(r) + \frac{b(\theta)}{r^2} + \frac{c(\phi)}{r^2\sin^2\theta}\right]$$

   Set up the Hamilton-Jacobi equation describing its motion and find its complete integral. [8]

   (b) Construct the action variable of a simple harmonic oscillator in one-dimension and hence obtain its fundamental frequency. [6]

$\frac{\partial W}{\partial q} = \sqrt{mk}\sqrt{\frac{2\alpha}{k} - x^2}$

$p = \frac{\partial H}{\partial \dot{q}}$ $J = \oint p\,dq$

300

4.(a) Consider a system, consisting of a free particle and a cluster of five particles, in three dimensions. The distance between all pairs in the cluster are fixed. If one particle in the cluster is fixed, find the number of degrees of freedom (NDF) for the system. Explain, how many of them would describe translational degrees of freedom for the system.

(2)

(b) Compute the rotational degrees of freedom of a rigid body in four dimensions (w, x, y, z). Explicitly, write down the rotation matrix when the angle of rotation is $\pi/4$ around the (yz)-symmetry plane in four dimensions.

(2)

(c) A rigid body, fixed in three dimensions, undergoes three successive rotations by angles $(\pi/2, \pi, \pi/2)$, respectively, along (z, x, z)-symmetry axes. Obtain the transformation matrix A resulted from three rotations using matrix operations. Find the symmetry axis and the angle of rotation, if any, under $\vec{X}' = A\vec{X}$.

Schematically, illustrate the finite rotations around all three symmetry axes. Do these transformations describe three independent Euler rotations? Give reasons.

If $P = \frac{1}{2}(1 + A)$, show that $P^2 = P$. Express $P\vec{X}$ in term of its components, i.e. $\vec{X} = (x, y, z)$.

(4+3+3)

5.(a) A rigid body fixed in three dimensions undergoes transformations, respectively, by Euler angles (ϕ, θ, ψ) around (z, x, z) space set of axes. If $\vec{\omega}$ denotes the instantaneous angular velocity of the rigid body, find the component $(\omega_{x_3}, \omega_{y_3}, \omega_{z_3})$ along its body set of axes (x_3, y_3, z_3) after three successive rotations.

(8) $\dot{\phi}$ $\dot{\theta}$

(b) Identify three Euler angles, in Q.5(a), under azimuth, polar and rotation angles. Name the angular velocities arised out of the Euler angles and explain their role to chracterise the motions of the rigid body.

Explain a steady precession of the rigid body.

When $\theta = 0$, how many independent Eulers rotations are there? Justify your answer using the result in Q.5(a).

(3+1+2)

6. Consider a torque free motion of a symmetrical top of mass M around its z-axis in three dimensions. The top is fixed at the origin with its body set of axes aligned along the principal axes.

(a) Use the operator identity : $\left(\frac{d}{dt}\right)_{\text{Space}} \equiv \left(\frac{d}{dt}\right)_{\text{Body}} + \left(\vec{\omega}\times\ \right)$

to explicitly write down the Euler equations of motion for the top in terms of the components of moment of inertia tensor and components of its instantaneous angular velocity.

Explain the origin and significance of the precession frequency Ω of the top and show that it is a constant of motion.

(4+4)

(b) Start with the definition of angular momentum $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = [I]\vec{\omega}$ and show that the tensor components I_{xi} of the moment of inertia I_{ij} are given by:

$$I_{xx} = M(r^2 - x^2)\ , \quad I_{xy} = -M(xy) \quad \text{and} \quad I_{xz} = -M(xz)\ .$$

(6)

$(\cos\theta + \sin\psi\sin\theta\,\dot{\phi} + \cos\psi\sin\theta)\dot{\phi}$

$\begin{pmatrix} 0 \\ 0 \\ \dot{\phi} \end{pmatrix}$