This question paper contains **4** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **48**

Unique Paper Code : **235466** **E**

Name of the Paper : **MAPT-404 : Differential Equations**

Name of the Course : **B.Sc. (H) Comp. Sc., B.Sc. (Appl. Phy. Sc.) Analytical Chemistry/Industrial Chemistry/B.Sc. Mathematical Science/B.Sc. Physical Science**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *two* parts from each question.

All questions are compulsory.

Marks are indicated against each question.

Unit I

1. (*a*) Solve : 6½

$$(y \sec^2 x + \sec x \tan x)\, dx + (\tan x + 2y)\, dy = 0.$$

(*b*) Solve : 6½

$$(x^2 + y^2 + 2x)\, dx + 2ydy = 0.$$

(*c*) Solve : 6½

$$xp^2 - 2yp + ax = 0.$$

P.T.O.

(*a*) Solve : 6½

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x} \sin x.$$

(*b*) Solve : 6½

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3,\ x > 0.$$

(*c*) Show that the Wronskian of two solutions of the second order homogeneous linear differential equation $a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)\,y = 0,$ is either identically zero or never zero on $a \leq x \leq b$, where a_0, a_1 and a_2 are continuous real functions on real interval $a \leq x \leq b$, and $a_0(x) \neq 0$ for any x on $a \leq x \leq b$. 6½

3. (*a*) Using method of variation of parameters, solve the differential equation : 6½

$$\frac{d^2y}{dx^2} + 4y = \sec^2 2x.$$

(*b*) Given that $y = x$ is a solution of 6½

$$(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution.

(*c*) A large tank initially contains 100 gal of pure water. Starting at $t = 0$, a brine containing 4 lb of dissolved salt per gallon flows into the tank at rate of 5 gal/min. The mixture is kept uniform by stirring, and the stirred mixture simultaneously flows out at the slower rate of 3 gal/min. 6½

(*i*) How much salt is in the tank at the end of 20 min ?

(*ii*) How much salt is present after a long time ?

4. (*a*) Solve : 6½

$$\frac{dx}{dt} + 4x + 3y = t,$$

$$\frac{dy}{dt} + 2x + 5y = e^t.$$

(*b*) Solve : 6½

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}.$$

(*c*) Solve : 6½

$$zy\,dx = zx\,dy + y^2\,dz.$$

Unit II

5. (*a*) Eliminate the arbitrary function f from the equation :

$$z = f\left(\frac{xy}{z}\right)$$

to form the corresponding partial differential equation. 5½

(*b*) Find the general solution of the differential equation : 5½

$$(y + x)\,px = (x + y)qy - (x - y)\,(2x + 2y + z).$$

(*c*) Find the complete integral of the equation : 5½

$$xp + 3yq = 2(z - x^2q^2).$$

6. (*a*) Find the complete integral of the equation : 6

$$(p^2 + q^2)y = z.$$

(*b*) Show that the equations : 6

$$xp = yq, \quad z(xp + yq) = 2xy$$

are compatible and find their solution.

(*c*) Reduce the equation : 6

$$\frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form.