[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 43 **E** **Your Roll No.....**

Unique Paper Code : 235266

Name of the Course : **B.Sc. (Hons.) Computer Science / B.Sc. Mathematical Sciences / B.Sc. Physical Sciences**

Name of the Paper : Calculus and Geometry (MAPT–202)

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the questions are compulsory.
3. Attempt any **two** parts from each question.
4. Marks of each part are indicated.

1. (a) Let f be a function defined on R by setting

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that f is derivable at x = 0, and that f' is not continuous at x = 0. (6)

(b) Show that the function f defined by $f(x) = x^2$ is uniformly continuous in $]-2, 2[$. (6)

(c) Use (ε, δ) definition to show that

$$\lim_{x \to 4} \sqrt{x} = 2 \quad (6)$$

2. (a) Show that the function f defined as follows :

$$f(x) = \begin{cases} \frac{|x-2|}{x^2-4}, & x \neq 2 \\ \frac{1}{4}, & x = 2 \end{cases}$$

is discontinuous at x = 2. State the kind of discontinuity. (6)

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(b) State and prove Lagrange's Mean Value Theorem and give its geometrical interpretation. (6)

(c) Find the asymptotes of the curve
$xy(x^2 - y^2) + 2x^2 + 2xy + 2y^2 - 1 = 0$. (6)

3. (a) Find the position and nature of multiple points of the curve
$$x^3 + y^3 + 3(x^2 + y^2) = 3xy + 1.$$
Also, find the equations of the tangents at each multiple point. (7)

(b) Examine the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ for concavity and convexity. Also, determine its points of inflexion. (7)

(c) Trace the curve $y^2(a + x) = x^2(3a - x)$. (7)

4. (a) Trace the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$. (6)

(b) If $I_{m,n} = \int x^m (\log x)^n dx$, show that

$(m + 1)I_{m,n} = x^{m+1}(\log x)^n - nI_{m,n-1}$. Find the value of $\int_0^1 x^4 (\log x)^3 dx$. (6)

(c) Find the area of a loop of the curve $r = a\sin 2\theta$. (6)

5. (a) Find the volume of the solid generated by rotating the ellipse $4x^2 + y^2 = 4$ about the x-axis. (6)

(b) Describe the graph of the equation $4y^2 - x^2 + 40y - 4x = -60$. (6)

(c) Find the equation of the ellipse whose foci are (2,1) and (2, –3) and major axis is of length 6. Also, write its reflection property. (6)

6. (a) Rotate the axes of coordinates to get rid of the xy-term from the equation $9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$ and trace the conic. (6.5)

(b) For $F = F(x,y,z)$, $G = G(x,y,z)$, $\varphi = \varphi(x,y,z)$, prove that
$\text{curl}(\varphi F) = \varphi\, \text{curl} F + \nabla\varphi \times F$. (6.5)

(c) (i) Find $\text{curl}\left(\text{curl}\,\vec{A}\right)$ if $\vec{A} = y^2x\,\hat{i} - 3yz\,\hat{j} + xy\,\hat{k}$. (3)

(ii) Find the center and radius of the sphere
$x^2 + y^2 + z^2 - 2x - 4y + 8z + 17 = 0$. (3 5)