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S. No. of Question Paper : **48**

Unique Paper Code : **235466**   **E**

Name of the Paper : **MAPT-404 : Differential Equations**

Name of the Course : **B.Sc. (H) Comp. Sc., B.Sc. (Appl. Phy. Sc.) Analytical Chemistry/Industrial Chemistry/B.Sc. Mathematical Science/B.Sc. Physical Science**

Semester : **IV**

Duration : **3 Hours**   Maximum Marks : **75**

Attempt *two* parts from each question.

All questions are compulsory.

Marks are indicated against each question.

Unit I

1. (*a*) Solve :   6½

   $$(y \sec^2 x + \sec x \tan x)\, dx + (\tan x + 2y)\, dy = 0.$$

   (*b*) Solve :   6½

   $$(x^2 + y^2 + 2x)\, dx + 2y\, dy = 0.$$

   (*c*) Solve :   6½

   $$xp^2 - 2yp + ax = 0.$$

2. (a) Solve :   6½

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}\sin x.$$

(b) Solve :   6½

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3, \; x > 0.$$

(c) Show that the Wronskian of two solutions of the second order homogeneous linear differential

equation $a_0(x)\dfrac{d^2y}{dx^2} + a_1(x)\dfrac{dy}{dx} + a_2(x)\,y = 0$, is either identically zero or never zero

on $a \le x \le b$, where a_0, a_1 and a_2 are continuous real functions on real interval

$a \le x \le b$, and $a_0(x) \neq 0$ for any x on $a \le x \le b$.   6½

3. (a) Using method of variation of parameters, solve the differential equation :   6½

$$\frac{d^2y}{dx^2} + 4y = \sec^2 2x.$$

(b) Given that $y = x$ is a solution of   6½

$$(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0,$$

find a linearly independent solution by reducing the order. Write the general

solution.

(c) A large tank initially contains 100 gal of pure water. Starting at $t = 0$, a brine containing

4 lb of dissolved salt per gallon flows into the tank at rate of 5 gal/min. The mixture

is kept uniform by stirring, and the stirred mixture simultaneously flows out at the slower

rate of 3 gal/min.   6½

(i) How much salt is in the tank at the end of 20 min ?

(ii) How much salt is present after a long time ?

4. (a) Solve : 6½

$$\frac{dx}{dt} + 4x + 3y = t,$$

$$\frac{dy}{dt} + 2x + 5y = e^t.$$

(b) Solve : 6½

$$\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}.$$

(c) Solve : 6½

$$zy\,dx = zx\,dy + y^2\,dz.$$

Unit II

5. (a) Eliminate the arbitrary function f from the equation :

$$z = f\left(\frac{xy}{z}\right)$$

to form the corresponding partial differential equation. 5½

(b) Find the general solution of the differential equation : 5½

$$(y + x)\,px = (x + y)qy - (x - y)\,(2x + 2y + z).$$

(c) Find the complete integral of the equation : 5½

$$xp + 3yq = 2(z - x^2q^2).$$

6. (a) Find the complete integral of the equation :   6

$$(p^2 + q^2)y = z.$$

(b) Show that the equations :   6

$$xp = yq, \qquad z(xp + yq) = 2xy$$

are compatible and find their solution.

(c) Reduce the equation :   6

$$\frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form.