This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8143**

Unique Paper Code : **235486** **D**

Name of the Paper : **Linear Algebra and Calculus**

Name of the Course : **B.A. (Hons.) (For Economics Hons.)**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all, selecting *two* questions from each Section.

Section I

1. (*a*) Let

$$R^2 = \{(a_1, a_2) : a_1, a_2 \in R\}.$$

Prove that R^2 is a vector space over R with addition and scalar multiplication defined as :

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$$

$$\alpha(a_1, a_2) = (\alpha a_1, \alpha a_2) \text{ where } (a_1, a_2), (b_1, b_2) \in R^2, \alpha \in R.$$ 6

(*b*) Define basis of a Vector space. Prove that the following subset S of R^3 is a spanning set of R^3, but not a basis of R^3,

$$S = \{(2,2,3), (-1,-2,1), (0,1,0)\}.$$ 6

(*c*) If A and B are symmetric matrices, show that AB + BA is symmetric and AB – BA is skew symmetric. 4

2. (*a*) State rank-nullity Theorem. Let $T: R^3 \to R^2$ be a linear transformation given by :

$$T(1,0,0) = (1,0), \ T(0,1,0) = (2,-1) \text{ and } T(0,0,1) = (4,3).$$

Find T(2, –3, 5). 6

(*b*) Consider the vectors :

$$v_1 = (1,0,1,2), \ v_2 = (0,1,1,2), \ v_3 = (1,1,1,3) \text{ in } R^4.$$

Is $S = (v_1, v_2, v_3)$ linearly dependent or linearly independent ? 6

(*c*) Using inner product, compute the angle between the diagonal of the cube and one of its sides. 4

3. (*a*) Let $T : R^3 \to R^3$ be a linear operator defined by :

$$T(x, y, z) = (x + y + z, y - z, 2x + z).$$

Find the standard matrix A representing T and verify T(X) = AX. 8

(*b*) Define an orthonormal basis of R^3. Prove that the set :

$$\left\{\left(\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}\right), \left(\frac{1}{3}, \frac{-2}{3}, \frac{-2}{3}\right)\right\}$$

form an orthonormal set in R^3 with respect to standard inner product. 8

Section II

4. (*a*) Use ε-δ definition to prove that the following function is continuous at $x = 0$.

$$f(x) = \begin{cases} x\cos\frac{1}{x} & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}$$ 6

(*b*) Determine the points of discontinuity of the following function :

$$f(x) = |x| + [x], \quad x \in R.$$ 6

5. (*a*) State the Lagrange's Mean Value theorem and use it to prove that for all x, y in R :

$$|\sin x - \sin y| \le |x - y|.$$ 6

(*b*) Let

$$f(x) = \begin{cases} \sin x & \text{if } x \le c \\ ax + b & \text{if } x > c \end{cases}$$

Find values of a and b (in terms of c) such that $f'(c)$ exist. 6

6. (*a*) State and check the validity of Intermediate value theorem for the function :

$$f(x) = \begin{cases} 1 - x & \text{if } 0 \le x \le 1 \\ -1 - x & \text{if } -1 \le x < 0 \end{cases}$$ 6

(*b*) Let f be a continuous function on $[a, b]$ and $X_1, X_2, \ldots\ldots\ldots, X_n$ be points of $[a, b]$. Show that there exist a point $c \in [a, b]$ such that :

$$f(c) = \frac{f(x_1) + f(x_2) + \ldots\ldots\ldots + f(x_n)}{n}.$$ 6

Section III

7. Locate all relative extrema and saddle point of the following functions : 9½

(i) $f(x, y) = 3x^2 - 2xy + y^2 - 8y$

(ii) $f(x, y) = x^4 + y^4 - 4xy$.

Also find their relative extremes value.

8. Let

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x^3 + y^6} & \text{if} \quad (x, y) \neq (0, 0) \\ 0 & \text{if} \quad (x, y) = (0, 0) \end{cases}$$

show that $f(x, y)$ has directional derivatives in all directions at (0, 0) but is not continuous at that point. 9½

9. Using Taylor's theorem, expand $xy^2 + 2x - 3$ in the powers of $x - 2$ and $y + 1$. 9½