This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8142**

Unique Paper Code : **235485** **D**

Name of the Paper : **Elements of Analysis [MT-III]**

Name of the Course : **B.A. (Hons.) Economics (Part II)**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are **3** Sections.

Attempt *all* the Sections.

Marks are indicated against each question.

Section I

Attempt any *three* questions.

1. (*a*) State and prove the Archimedean property of real numbers. 5

(*b*) Define the supremum and infimum of a set S of real numbers. Find the supremum and infimum of the following set : 5

$$S = \left\{1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{2^2}, ..., 1 + \frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^{n-1}}, ...\right\}.$$

P.T.O.

2. (*a*) State Cauchy's general principle of Convergence. Use this to show that the sequence (a_n), where : 5

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}$$

is not convergent.

(*b*) Show, by definition : 5

$$\lim_{n\to\infty} \sqrt[n]{n} = 1.$$

3. (*a*) If (a_n) and (b_n) are two convergent sequences with : 5

$$\lim(a_n) = a, \lim(b_n) = b,$$

show that sequence $(a_n + b_n)$ is also convergent and :

$$\lim(a_n + b_n) = (a + b).$$

(*b*) Show that : 5

$$\lim_{n\to\infty} \frac{(1 + y)^n}{n!} = 0$$

for all y.

4. (*a*) Define monotonic sequence. Define (a_n) as :

$$a_1 = 8, \ a_{n+1} = 2 + \frac{1}{2}a_n.$$

Show that (a_n) is monotonic and bounded. Also find its Limit. 5

(*b*) State Cauchy's 2nd theorem on limits. Prove that : 5

$$\left\{\frac{(2n)!}{(n!)^2}\right\}^{\frac{1}{n}} = 4.$$

Section II

Attempt any *two* questions.

5. (*a*) State Ratio test for the positive term series. 2.5

(*b*) Show that the series : 5

$$1 + \frac{1}{2!} + \frac{1}{3!} + \dots\dots$$

is convergent.

(*c*) Test for the convergence : 5

$$\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \dots\dots\dots$$

6. (*a*) Does the series :

$$\sum \cos\frac{1}{n}$$

converge ? Justify. 2.5

(*b*) Test for the convergence of the series :

(*i*) $\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n + 1}$ 5

(*ii*) $\sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}$. 5

7. (*a*) State Leibnitz test for the convergence of Infinite series. 2.5

(*b*) Test for the convergence and absolute convergence of the following series :

(*i*) $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \ldots\ldots\ldots$ 5

(*ii*) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\log n}$. 5

Section III

Attempt any *two* questions.

8. Determine the radius of convergence of the following power series : 5+5

(*i*) $\sum_{n=1}^{\infty} \frac{n!^2 x^n}{(2n)!^2}$

(*ii*) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n}$.

9. (*a*) Write down the power series expansion for cos x. 5

(*b*) Prove the identity : 5

$$C^2(x) + S^2(x) = 1,$$

for all $x \in$ IR, where S(x), C(x) denote the Sine and Cosine functions respectively.

10. (*a*) Prove that if R is the radius of convergence of the power series :

$$\sum a_n x^n,$$

then the series is absolutely convergent if $|x| < R$. 5

(*b*) Show that : 5

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots, \quad -1 < x < 1.$$