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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8459**

Unique Paper Code : **235365** **C**

Name of the Paper : **MACT-302 (Mathematics-II)**

Name of the Course : **B.Sc. (Hons.) (Chemistry) Part II**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

This question paper has six questions in all. Attempt *two* parts from each question. *All* questions are compulsory.

Use of scientific calculator is allowed.

1. (*a*) Solve the boundary value problem : 6½

$$y''(t) - 2y'(t) + 2y(t) = 0,\ y(0) = 1,\ y\left(\frac{\pi}{2}\right) = 2.$$

(*b*) Solve the partial differential equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$ 6½

(*c*) Use the power-series method to solve the differential equation $y''(x) - y(x) = 0$ and show that the solution can be expressed in the form $c_1 e^{x} + c_2 e^{-x}$. 6½

P.T.O.

2. (a) Evaluate the integral of $f(x, y) = 1 + 2xy$ over the region E bounded by the line $y = x$ and the curve $y = x^2$. Also calculate the area of the region. 6½

(b) Find the value of the integral $\iint_D e^{-(x^2+y^2)}dxdy$ where D is the disk $x^2 + y^2 \leq a^2$. 6½

(c) Evaluate the integral of the function $f(r, \theta, \varphi) = r^2 \cos^2 \theta \sin^2 \varphi$ over a sphere of radius 'a' with centre at origin. 6½

3. (a) Determine the nature of the stationary points of the function : 5

$$f(x, y) = x^3 + 6xy^2 - 2y^3 - 12x.$$

(b) Show that the function $\cos(ax) \cos(by) \cos(cz)$ is an eigenfunction of the operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. What is the eigenvalue ? 5

(c) Determine whether the operators $\hat{A} = \frac{d}{dx}$ and $\hat{B} = \frac{d^2}{dx^2} + 2\frac{d}{dx}$ commute or not ? 5

4. (a) Show that :

$$\frac{d}{dt}(\vec{u} \cdot \vec{v}) = \frac{d\vec{u}}{dt} \cdot \vec{v} + \vec{u} \cdot \frac{d\vec{v}}{dt}$$

$$\frac{d}{dt}(\vec{u} \times \vec{v}) = \frac{d\vec{u}}{dt} \times \vec{v} + \vec{u} \times \frac{d\vec{v}}{dt}$$

where $\vec{u}$ and $\vec{v}$ are vectors in 3-dimensional space. 6½ (*b*) Show that : 6½

$$\nabla^2\left(\frac{x}{r^3}\right)=0,$$

where

$$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k} \text{ and } r=|\vec{r}|.$$

(*c*) Prove that div $\varphi\vec{v}=\varphi\vec{\nabla}\cdot\vec{v}+\vec{v}\cdot\vec{\nabla}\varphi$. Use this result to evaluate div $\varphi\vec{v}$ if

$\varphi=xy$ and $\vec{v}=3y^2\hat{i}+2xz\hat{k}$. 6½

5. (*a*) Solve the following set of equations :

$$x+2y+3z=-5$$
$$-x-3y+z=-14$$
$$2x+y+z=1$$

using Cramer's rule. 6½

(*b*) Find the eigenvalues and eigenvectors of $\mathbf{A}=\begin{pmatrix}-2 & 1 & 1\\ -11 & 4 & 5\\ -1 & 1 & 0\end{pmatrix}$. 6½

(*c*) State the condition under which a square matrix is invertible. Find the characteristic equation of the matrix $\begin{pmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{pmatrix}$ and hence compute its inverse if it exists. 6½

6. (*a*) Solve the equation $z^5 = 1$ and plot the roots in the complex plane. 6½

(*b*) Determine the region in the complex plane described by $2 < |z - 2 + i| \leq 4$ and explain. 6½

(*c*) Does matrix multiplication commute ? Justify with an example.

If A and B are two matrices such that AB = 0. Does it imply that A = 0 or B = 0 ? Justify with an example. 6½