[This question paper contains 4 printed pages.]

1943 *Your Roll No.* ....

B.Sc. (H) Computer Science / II Sem. **C**

Paper CS-204 : Probability

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **all** *questions.*
All *questions carry equal marks.*

1. The dice game craps is played as follows. The player throws two dice, and if the sum is seven or eleven, then she wins. If the sum is two, three or twelve, then she loses. If the sum is anything else, then she continues throwing until she either throws that number again (in which case she wins) or she throws a seven (in which case she loses). Calculate the probability that the player wins.

2. Suppose that each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. What is the probability that none of the three men select his own hat ?

P.T.O.

3. Suppose that two teams are playing a series of games, each of which is independently won by team A with probability p and by team B with probability 1-p. The winner of the series is the first team to win four games. Find the expected number of games that are played.

4. State and prove Markov's inequality.

5. Let X be the number of times that a fair coin, flipped 40 times, land heads. Find the probability that $X = 20$. Use the normal approximation and then compare it with exact solution.

6. Let the probability density of X be given by :

$f(x) = c\ (4x - 2x^2),\ 0 < x < 2$ else,

$= 0;$

What is the value of c and evaluate

$P\{1/2 < X < 3/2\}$.

7. If X is a non negative integer valued random variable, show that

$$E(X) = \sum_{n=0}^{\infty} P(X > n)$$

8. If X and Y are independent Poisson random variables with respective means m_1 and m_2 calculate the conditional expected value of X given that $X + Y = n$.

9. Sam will read either one chapter of his probability book or one chapter of his history book. If the number of misprints in a chapter of his probability book is Poisson distributed with mean 2 and of number of misprints of his history chapter is Poisson distributed with mean 5, then assuming Sam is equally likely to choose either book, what is the expected number of misprints that Sam will come across ?

10. A particle moves on a circle through points which have been marked 0, 1, 2, 3, 4 (in a clockwise order). At each step it has a probability p of moving to the right (clockwise) and 1-p to the left (counter clockwise). Let X_n denote its location on the circle after the n^{th} step. The process $\{X_n, n \geq 0\}$ is a Markov chain.

(a) Find the transition probability matrix.

(b) Calculate the limiting probabilities.

11. Give an algorithm for simulating a random variable having density function

$f(x) = 30\ (x^2 - 2x^3 + x^4), \quad 0 < x < 1.$

12. Show that maximum entropy for the ensemble

$$X = \{(x_1, p_1), (x_2, p_2), \ldots\ldots\ldots\ldots (x_n, p_n)\}$$

is achieved when $p_1 = p_2 = \ldots\ldots\ldots\ldots = p_n = 1/n$.

P.T.O.

13. Suppose that the probability of a dry day (state 0) following a raining day (state 1) is 1/3 and that the probability of a raining day followed by a dry day is 1/2. Given that May 1 is a dry day, what is the probability that May 3 is a dry day.

14. Suppose X is a Poisson random variable with mean m. The parameter m is itself a random variable whose distribution is exponential with mean 1. Show that

$$P\{X = n\} = (1/2)^{n+1}$$

15. Let X_1 .... X_n be independent and identically distributed random variables, each with mean μ and variance σ^2. Show that $E[S^2] = \sigma^2$, where

$$S^2 = \sum_{i=1}^{n} \frac{\left(X_i - \overline{X}\right)^2}{n-1}$$