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Your Roll No. ....

6615

B.Sc. (Hons.) **B**

COMPUTER SCIENCE/III Sem.

Paper CS-303—Algebra

(Admissions of 2010 and onwards)

Time : 3 Hours *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the questions are compulsory.

All questions carry equal marks.

1. Define a group and show that :

$$GL_2(\mathbf{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a,\ b,\ c \text{ and } d \in \mathbf{R} \text{ with } ad \neq bc \right\}$$

in a group w.r.t. ordinary matrix multiplication. 5

2. State Fermat's Little theorem and use it in conjunction with computers to test primality of $2^{257} - 1$, giving algorithms in steps. 5

3. Determine whether the set S is a subring of the ring of complex numbers C or not, $S = \{ x + iy \mid x, y \in \mathbf{Z} \}$. 5

P.T.O.

4. Define a field and show that the ring of Gaussian integers $Z + iZ = \{x + iy \mid x, y \in Z\}$ is not a field, however $Q + iQ$ is. 5

5. (*i*) Show that $S = \{t_1 v_1 + t_2 v_2 \mid 0 \le t_1, t_2 \le 1$ and v_1, v_2 are fixed vectors of a vector space over reals$\}$ is a convex set. 3

(*ii*) Find the rank of matrix :

$$\begin{pmatrix} 1 & 2 & 7 \\ 2 & 4 & -1 \end{pmatrix}.$$ 2

6. Prove that $W = \{(x, 2x) \mid x \in \mathbf{R}\}$ is a subspace of $\mathbf{R}^2$ ($\mathbf{R}$). Show it geometrically in $\mathbf{R}^2(\mathbf{R})$. 5

7. What is the dimension of the following spaces : 5

(*a*) Symmetric 3×3 matrices over reals.

(*b*) Lower triangular 3×3 matrices over reals. Also find a basis in each of the above cases.

8. Let M be the space of all $n \times n$ matrices. Let $P : M \to M$ be the map such that $P(A) = \dfrac{A + A'}{2}$. Show that P is linear and kernel of P is the space of skew-symmetric matrices. 5

9. Find the matrix associated with the following linear map $F : \mathbf{R}^4 \to \mathbf{R}^2$ given by $F(x_1, x_2, x_3, x_4) = (x_1, x_2)$. 5

10. State and prove Bessel's inequality. 5

11. Find orthonormal basis for the subspace of $\mathbf{R}^4$ generated by $(1, 1, 0, 0)$, $(1, -1, 1, 1)$ and $(-1, 0, 2, 1)$. 5

12. Let A be a diagonal matrix $\begin{pmatrix} a_1 & 0 \ldots\ldots & 0 \\ 0 & a_2 \ldots\ldots & 0 \\ \vdots & & \vdots \\ 0 & & a_n \end{pmatrix}$

   (*a*) What is the characteristic polynomial of A ?

   (*b*) What are its eigenvalues ? 5

13. Find the maximum and minimum of the function :

   $f(x, y) = 3x^2 + 5xy - 4y^2$ on the unit circle. 5

14. Classify and skeh the curve : $2xy - y^2 = 1$. 5

15. Let $(A; \leq)$ be poset. Let $\leq_R$ be a binary relation on A such that for a, b in A $a \leq_R b$ iff $b \leq a$.

   Show that if $(A, \leq)$ is a lattice then so is $(A, \leq_R)$. 5