[This question paper contains 4 printed pages.]

6588 *Your Roll No.* ....

B.Sc. (Hons.) Computer Science / I Sem. B

Paper – CS 103 : Calculus – I

(Admissions of 2001 to 2009)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **all** *questions.*
All *questions carry equal marks.*
Use of Scientific Calculator is allowed.

1. (a) Evaluate $\lim_{x \to \frac{\pi}{4}} (\tan x)^{\tan 2x}$.

   (b) Find the absolute extrema values of the function $g(x) = 8x - x^4$ on $[-2, 1]$. (5)

2. (a) State the Intermediate value theorem for real-valued continuous functions. Can the hypothesis of continuity be dropped in the theorem ? Justify.

   (b) Show that the equation $x^3 - 15x + 1 = 0$ has all three solutions in the interval $[-4, 4]$. (5)

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3. Graph the function $y = x^{2/3}\left(\frac{5}{2} - x\right)$. Include the coordinates of any local extreme points and inflection points. (5)

4. Solve the following initial value problem for $\vec{r}$ as a vector function of t :

$$\frac{d^2\vec{r}}{dt^2} = -\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$\vec{r}(0) = 10\hat{i} + 10\hat{j} + 10\hat{k}$$

$$\left.\frac{d\vec{r}}{dt}\right|_{t=0} = \vec{0}$$ (5)

5. (a) For any two real numbers x and y, prove that

$$|\sin x - \sin y| \leq |x - y|$$

(b) If f is a real-valued function satisfying $f'(x) = 2x$ for all x and $f(-2) = 3$, find $f(2)$. (5)

6. Assuming the validity of differentiation under integral sign, prove that

$$\int_{\frac{\pi}{2}-\alpha}^{\frac{\pi}{2}} \sin\theta \cos^{-1}(\cos\alpha \ \mathrm{cosec}\theta)\, d\theta = \frac{\pi}{2}(1 - \cos\alpha)$$

(5)

7. Evaluate the following limits (if they exist) :

(i) $\lim_{x \to \infty} \frac{\sin x + \cos x}{x^4}$

(ii) $\lim_{x \to 0} x \sin \frac{1}{x}$ (5)

8. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin. (5)

9. Use Taylor's formula to find a quadratic approximation of $e^x \sin y$ at the origin. Estimate the error in the approximation if $|x| \le 0.1$ and $|y| \le 0.1$. (5)

10. Find the absolute maxima and minima of the function $f(x, y) = x^2 - xy + y^2 + 1$ on the closed triangular plate in the first quadrant bounded by the lines $x = 0$, $y = 4$, $y = x$. (5)

11. Prove that a sequence $\langle a_n \rangle$ converges to 0 if and only if the sequence of absolute values $\langle |a_n| \rangle$ converges to 0. (5)

12. (a) Test for convergence the infinite series :

$$\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \ldots\ldots$$

(b) Find the value of b for which

$$1 + e^b + e^{2b} + e^{3b} + \ldots\ldots = 9$$ (5)

13. (a) Define an absolutely convergent series. Show that every absolutely convergent series is convergent. Is the converse true ?

(b) Estimate the magnitude of the error involved in using the sum of the first four terms to approximate the sum of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.01)^n}{n}$. (5)

14. Find the directions in which the function $f(x, y, z) = \ln xy + \ln yz + \ln xz$ increases and decreases most rapidly at (1, 1, 1). Also find the derivative of the function in these directions. (5)

15. (a) Show that the function

$$f(x, y) = \frac{x^2}{x^2 - y}$$

has no limit as $(x, y) \to (0, 0)$.

(b) The plane $x = 1$ intersects the paraboloid $z = x^2 + y^2$ in a parabola. Find the slope of the tangent to the parabola at (1, 2, 3). (5)