[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 6013 **D** **Your Roll No...............**

Unique Paper Code : 235365

Name of the Course : **B.Sc. (Hons.) Chemistry**

Name of the Paper : Mathematics II / Code : MACT–302

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt **two** parts from each question.
4. **All** questions are compulsory.
5. Use of scientific calculator is allowed.

1. (a) Solve the boundary value problem $y''(x) + 8y'(x) + 16y(x) = 0$, $y(0) = 1$, $y(1) = 1$. (6½)

   (b) Solve the classical wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$, When the separation constant is (i) zero (ii) greater than zero. (6½)

   (c) Use the power-series method to solve the differential equation $y''(x) - 9y(x) = 0$ and show that the solution can be expressed in the form $c_1e^{3x} + c_2e^{-3x}$, where c_1 and c_2 are arbitrary constants. (6½)

2. (a) Show that $\int_0^1 dx \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dy = \int_0^1 dy \int_0^1 \frac{x^2 - y^2}{x^2 + y^2} dx$. (6½)

   (b) Evaluate the double integral $\iint_D e^{-(x^2+y^2)} x^2 \, dxdy$ where D is the disk $x^2 + y^2 \le a^2$. (6½)

   (c) Change the order of integration in the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} xy^2 \, dydx$ and then find the value of the integral (sketch the region of integration). (6½)

3. (a) Determine the nature of the stationary points of the function $f(x, y) = 3x^2 + 12x - 6y^2 + 4y^3 + 5$. (5)

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(b) Show that the function Sin(αx)Sin(βy)Sin(γz) is an eigen function of the operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. What is the eigen value ? (5)

(c) Determine whether the operators $\hat{A} = \frac{d}{dx} - x$ and $\hat{B} = \frac{d}{dx} + x$ commute or not. (5)

4. (a) If $\vec{A} = t^3\hat{i} + 2t\hat{j} + e^t\hat{k}$ and $\vec{B} = \operatorname{Sin} t\,\hat{i} - \operatorname{Cos} t\,\hat{j} + t\hat{k}$, then evaluate

(i) $\frac{d}{dt}\left(\vec{A}.\vec{B}\right)$ (ii) $\frac{d}{dt}\left(\vec{A}\times\vec{B}\right)$ (6½)

(b) If φ = In r, show that $\nabla\varphi = \frac{\vec{r}}{r^2}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = \left|\vec{r}\right|$. (6½)

(c) Show that div v = 0 if v = curl w. (6½)

5. (a) Solve the following equations and find the value of x, y, z

$x + y + z = 7$

$2x - 2y + 3z = 14$

$x - y + z = 1$ (use Cramer's rule) (6½)

(b) Find the eigenvalues and eigenvectors of $\begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$. (6½)

(c) State the condition under which a square matrix is invertible. Find the characteristic equation of the matrix $\begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}$ and hence compute its inverse. (6½)

6. (a) Solve the equation $z^5 = -1$ and plot the roots in the Complex plane. (6½)

(b) Find all non-zero complex number z satisfying $z^2 + |\bar{z}| = 0$. (6½)

(c) Define unitary matrix. Show that the determinant of an orthogonal matrix is equal to ±1. If det(A) = 0.2 then find the value of $\det(BA^{-1}B^{-1})$.

$\left(\text{Use } \det\left(A^{-1}\right) = 1/\det(A)\right)$ (6½)