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Your Roll No. ....

B.Sc. (Hons.) / V Sem./Computer Sc. A

Paper 504 : Numerical Analysis and Scientific Computing

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **all** questions. Parts of a question must be answered together. Use of non-programmable scientific calculator is allowed.*

1. (a) Construct the taylor series for the function

$f(x) = \sin^{-1} x$, $|x| < 1$

and bound the error when truncated after n terms. 5

(b) Assume that $x_A = 0.786$ has three significant digits with respect to x_T. Bound the relative error in x_A. For $f(x) = \frac{1}{\sqrt{1-x}}$ bound the error and relative error in $f(x_A)$ with respect to $f(x_y)$. 3

2. (a) Assume $f(x)$ is twice continuously differentiable as $[a, b]$, $f(a) < 0$, $f(b) > 0$, $f'(x) > 0$ and $f''(x) < 0$, $\forall x \in [a, b]$. Then prove that the iterates x_n are strictly increasing to α and iterates z_n are strictly decreasing to α. Also, show that

$$\lim_{n \to \infty} \frac{x_{n+1} - z_{n+1}}{(x_n - z_n)^2} = \frac{f''(\alpha)}{2f'(\alpha)}.$$ 5

(b) Apply Newton's method to the function

$$f(x) = \begin{cases} \sqrt{x-a} & x \geq a \\ -\sqrt{a-x} & a \geq x \end{cases}$$

with the root $x = a$. What is the behavior of the iterates ? Do they converge and if so, at what rate ? 5

3. (a) Consider –h, o, h as nodal points and $\in$ the maximum value of the rounding error in the function evaluations. Show that the effect of these rounding errors on the quadratic interpolation error is bounded by 1.25 $\in$ for $-h \leq x \leq h$. 4

(b) Prove that $f[x_0, x_1, \ldots x_n] = \frac{f^n(l_n)}{\underline{|n}}; \forall n$

Where $l_y \in X\{x_0, x_1 \ldots x_n\}$ and $X\{x_0, x_1, \ldots x_n\}$ is the smallest closed internal containing $x_0, x_1, \ldots, x_n$. 6

4. (a) Obtain the asymptotic error formula for Simpson's 1/3 rule. 5

(b) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$

using trapezoidal rule by taking $n = 4$.

Evaluate the actual error and an upper bound of the error. 4

(c) Evaluate the integral $I = \int_1^2 xe^{-x^2}\,dx$

using Gaussian Quadrature ($n = 2$). Compare this solution with exact solution. 3+1

5. (a) Find the interval which contains the eigen values of the symmetric matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$

using Gerschgorin bounds. 4

(b) Find the linear least square approximation to $f(x) = \sin x$ on $\left[0, \frac{\pi}{2}\right]$ with respect to the weight function $W(x) = 1$. Compare the error with linear Taylor's polynomial about $x_0 = \pi/4$. 3+1

(c) Write the codes in MATLAB/MATHEMATICA/ MAPLE to solve the following integral using Simpson's 1/3 rule $\int_1^2 (x^2 + x)dx$. 3

6. (a) Solve the following system of equations by using Jalobi method

$6x_1 - 2x_2 + x_3 = 11$

$x_1 + 2x_2 - 5x_3 = -1$

$-2x_1 + 7x_2 + 2x_3 = 5$

(Perform 3 iteration) with $X_0 = [0, 0, 0]'$. 5

(b) Given $A = \begin{bmatrix} 5 & -5 & 7 \\ -4 & 2 & -4 \\ -7 & -4 & 5 \end{bmatrix}$

Calculate Frobenius norm $\|A\|_f$, $\|A\|_\infty$ and $\|A\|_1$ 3

7. (a) Use Rayleigh-Ritz method to approximate the solution of

$y^{11} = 3x + 1,$

$y(0) = 0$

$y(1) = 0$

using a quadratic in x as the approximating function. 5

(b) Determine the value of y where $x = 0.1$ & $x = 0.2$ given $y^1 = -2x - y$

$y(0) = -1$

using modified Euler's method. 4

(c) For the differential equation $\frac{dy}{dx} = 1 + y^2$ satisfying $y(0) = 0$, calculate $y(0.2)$, $y(0.4)$ and $y(0.6)$ using Euler's method. Using these values evaluate $y(0.8)$ with the held of Tdam-Moulton method. 6