This question paper contains 4 printed pages.]

1778 *Your Roll No.* ....

B.Sc. (Hons.) / I Sem./Computer Sc. A

Paper 103 – Calculus–I

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **all** questions. All questions carry equal marks.*

Use of scientific calculator is allowed.

1. If $f(x) = x^2 + 1$, $x_0 = 1$, $L = 2$, $\in = 0.1$, find $\delta > 0$, satisfying, $|f(x) - L| < \in$, whenever $0 < |x - x_0| < \delta$.

2. If $f(x) = \begin{cases} x+a & x<1 \\ b & x=1 \\ \dfrac{a}{x+1} & x>1, \end{cases}$

   find values of a, b for which $f(x)$ is continuous.

3. Verify the hypothesis and conclusion of Rolle's Theorem for the function $f(x) = \sin^2 x + \sin x$, $0 \leq x \leq \pi$.

4. Obtain Mclaurim's series expansion of

$$f(x) = \log(1+x).$$

5. Prove that.

$$\lim_{n\to\infty} n^{1/n} = 1 \text{ and } \lim_{n\to\infty} \frac{1+2^{1/2}+\ldots+n^{1/n}}{n} = 1.$$

6. State Cauchy Integral Test for convergence of a positive term series. Apply it to discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p} \ (p>0)$.

7. Discuss the Absolute and conditional convergence of the series

(i) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n-1}$

(ii) $\sum_{n=1}^{\infty} \frac{x^n}{n!} (x>0)$.

8. For what value of α, will :

$$\lim_{x\to 0} \frac{\tan \alpha^2 x + 8 \tan \alpha x}{\sin 4x} = 1 \ ?$$

9. Graph the equation :

$y = 5x^{2/5} - 2x.$

Include the coordinates of any local extreme points and inflection points.

10. The velocity of a particle moving in space is

$$\frac{d\vec{r}}{dt} = (\cos t)\,\hat{i} - (\sin t)\,\hat{j} + \hat{k}.$$

Find the particle's position as a function of t if $\vec{r} = 2\hat{i} + \hat{k}$ when $t = 0$.

11. Assuming the validity of differentiation under integral sign, evaluate the following integral :

$$\int_0^{\infty} e^{-\alpha x} \frac{\sin \beta x}{x}\, dx \quad (\alpha \geq 0).$$

12. (a) Show that the function $f(x, y) = \dfrac{x^4}{x^4 + y^2}$ has no limit as $(x, y) \to (0, 0)$.

(b) Find $\dfrac{\partial^2 w}{\partial x \partial y}$ if $w = xy + \dfrac{e^y}{y^2 + 1}$.

13. Obtain the linearization $L(x, y, z)$ of the function

$f(x, y, z) = xz - 3yz + 2$

at the point (1, 1, 2). Also find an upper bound for the magnitude of the error E in the approximation $f(x, y, z) \approx L(x, y, z)$ over the region

$R : |x-1| \leq 0.01, |y - 1| \leq 0.01, |z - 2| \leq 0.02.$

14. Find the points on the curve $x^2 + xy + y^2 = 1$ in the xy-plane that are nearest to and farthest from the origin.

15. Find the directions in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$

(a) increases most rapidly at the point (1, 1).

(b) decreases most rapidly at the point (1, 1).

(c) What are the directions of zero change in f at (1, 1) ?