[This question paper contains 6 printed pages.]

1248 *Your Roll No.* ....

B.Sc. (Hons.) / I **A**

CHEMISTRY – Paper IV

(Mathematics – I)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Six** *questions in all, selecting at least* **one** *question from each Section.*

SECTION A

1\. (a) If $y = \left[x + \sqrt{1 + x^2}\right]^m$, prove that

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0 \qquad (3)$$

(b) If $u = \sin^{-1}\frac{x^2 + y^2}{x + y}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u \qquad (3)$$

(c) Obtain Maclaurin's series expansion of $\sin x$.

(4)

P.T.O.

2. (a) Evaluate :

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}} \quad (3)$$

(b) Find the equation of the tangent to the parabola $y^2 = 4x + 5$ parallel to the line $2x - y = 3$. (3)

(c) Discuss the validity of Rolle's theorem for $f(x) = (x-a)^m(x-b)^n$ in $[a, b]$; m, n being positive integers. (3)

3. (a) Show that the radius of curvature at any point of the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ is equal to three times the length of the perpendicular from the origin to the tangent. (3)

(b) Find the asymptotes of the curve :

$$x^3 + x^2y - xy^2 - y^3 - 2x^2 + 2y^2 + x + y + 1 = 0.$$

(3)

(c) Trace the curve :

$$y^2(a + x) = x^2(3a - x) \quad (3)$$

SECTION B

4. (a) Evaluate the following :

(i) $$\int_0^{\pi/2} \frac{\sin^2 x}{1 + \sin x \cos x}\,dx$$

(ii) $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx$ (5)

(b) If $U_n = \int_0^{\pi/2} \theta \sin^n\theta \, d\theta \ (n > 1)$, prove that

$$U_n = \frac{n-1}{n} U_{n-2} + \frac{1}{n^2}.$$ Deduce that $U_5 = \frac{149}{225}$.

(4)

5. (a) Find the area of the loop of the curve

$x(x^2 + y^2) = a(x^2 - y^2)$. (4)

(b) Show that the length of the loop of the curve

$3ay^2 = x(x - a)^2$ is $\frac{4a}{\sqrt{3}}$. (5)

6. (a) Solve any **two** of the following differential equations :

(i) $\left(x + 2y^3\right)\frac{dy}{dx} = y$

(ii) $[y(1+x^{-1}) + \sin y]dx + [x + \log x + x\cos y]dy = 0$

(iii) $\left(x^2 + y^2\right)\frac{dy}{dx} = xy$ (4)

(b) Solve any **two** of the following differential equations :

(i) $y = 2x\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^3$

(ii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \left(e^x + 1\right)^2$

(iii) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = \sin 2x$ (5)

SECTION C

7. (a) Find the value of λ so that the equation $2x^2 + xy - y^2 - 11x - 5y + \lambda = 0$ may represent a pair of straight lines. (3)

(b) Show that the equation of the circle drawn on the chord $x\cos\alpha + y\sin\alpha - p = 0$ of the circle $x^2 + y^2 = a^2$ as diameter is $x^2 + y^2 - a^2 - 2p(x\cos\alpha + y\sin\alpha - p) = 0$. (3)

(c) Prove that the locus of the middle points of focal chords of the parabola $y^2 = 4ax$ is $y^2 = 2a(x-a)$. (3)

8. (a) Show that the locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^6}{x^2} + \frac{b^6}{y^2} = \left(a^2 - b^2\right)^2$. (4)

(b) Prove that the line $lx + my = n$ will touch the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2l^2 - b^2m^2 = n^2$. (5)

9. (a) Show that $12x^2 - 2y^2 - 6z^2 - 2xy + 7yz + 6zx = 0$ represents a pair of planes. Find the angle between the planes. (3)

(b) Find the magnitude and the equations of the line of shortest distance between the lines :

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}; \ \frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$$ (3)

(c) Find the centre and the radius of the circle :

$x + 2y + 2z = 15,\ x^2 + y^2 + z^2 - 2y - 4z = 11.$ (3)

SECTION D

10. (a) Prove that :

$$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$$ (3)

(b) Solve : $z^6 - z^5 + z^4 - z^3 + z^2 - z + 1 = 0.$ (3)

(c) Find the modulus and argument of

$$\left(\frac{1+\cos\theta + i\sin\theta}{1+\cos\theta - i\sin\theta}\right)^n,$$

n is a positive integer and $\theta \neq (2K+1)\pi$. (3)

11. (a) Solve the equation : $3x^3 + 14x^2 - 28x - 24 = 0$, the roots being in G.P. (3)

(b) Form the cubic whose roots are the values of α, β, γ given by the relations :

$$\alpha + \beta + \gamma = 3,\ \alpha^2 + \beta^2 + \gamma^2 = 5,\ \alpha^3 + \beta^3 + \gamma^3 = 11.$$

(3)

(c) If α, β, γ be the roots of $x^3 + qx + r = 0$, form an equation whose roots are $(\beta - \gamma)^2$, $(\gamma - \alpha)^2$, $(\alpha - \beta)^2$. (3)