This question paper contains 4 printed pages.]

1777 *Your Roll No. ....*

B.Sc. (Hons.) Computer Science / I Sem. A

Paper-102 : Discrete Structures

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

***All** questions are compulsory. Attempt all the parts of a question together.*

1. (a) A palindrome is a word that reads the same forward or backward. How many seven-letter palindromes can be made out of English alphabet ?

3

(b) In how many ways can 20 boys and 7 girls stand in a circle so that no two girls are next to each other.

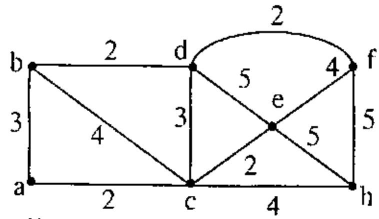
3

2. (a) Prove that a graph with e = V – 1 that has no circuit is a tree. Note that 'e' represents the number of edger and 'v' stands for the number of vertices.

3

(b) Find the minimum spanning tree for the graph :

[P.T.O.



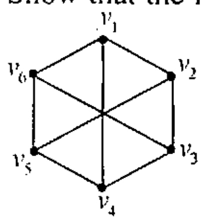
Show all the steps. 4

(c) Construct an optimal binary prefix code tree for the following frequency distribution :

3, 5, 7, 14, 25, 50, 70 3

3. (a) Does K_{13}, a complete graph with 13 vertices have an Eularian Circuit ? Justify. 2

(b) A graph is said to be r-colored if it requires 'r' number of colors to color all its vertices so that every pair of adjacent vertices are colored with different colors. Show that the following graph is 2-colorable : 3



4. (a) Use the simple algebraic operations on the sums to prove :

$$\sum_{i=0}^{n} \sum_{j=0}^{i} a_i a_j = \frac{1}{2}\left(\left(\sum_{i=0}^{n} a_i\right)^2 + \left(\sum_{i=0}^{n} a_i^2\right)\right)$$ 4

(b) Prove or disprove :

(i) $(2x-1) \leq \lceil x \rceil + \lfloor x \rfloor \leq (2x+1) \forall x \in \mathbb{R}$ 3

(ii) $x \leq \lceil x \rceil + \lfloor x \rfloor \leq 3x \forall\, x \geq 1$ 1

5. (a) Every particle inside a nuclear reactor splits into two particles in each second. Suppose one particle is injected into the reactor every second beginning at $t = 0$. How many particles are there in the reactor at the nth second ? 3

(b) Let 'a' be a numeric function such that

$$a_r = \begin{cases} 2 & 0 \le r \le 3 \\ 2^{-r} + 5 & r \ge 4 \end{cases}$$

(i) Determine $S^2 a$ (ii) Determine $S^{-2} a$ 4

6. (a) Determine the solution for the given recurrence relation using the method of generating functions only :

$a_r = a_{r-1} + a_{r-2}$

Where $a_0 = 0,\ a_1 = 2,\ a_2 = 3$

and the relation is valid for $r \ge 3$. 5

(b) Given that $a_0 = 0$, $a_1 = 1$, $a_2 = 4$, $a_3 = 12$, satisfy the given recurrence relation :

$a_r + c_1 a_{r-1} + c_2 a_{r-2} = 0$,

Determine a_r. 3

7. (a) Prove that :

$$\big(\forall x\,(P(x) \to Q(x)) \wedge \forall x\,(Q(x) \to S(x))\big) \to \forall x\,(P(x) \to S(x))$$

4

(b) Prove the conclusion using the theory of inference for predicate calculus :

Everyone in this college has purchased a computer. Hari is a student in this college. This implies that Hari has purchased a computer too. 5

(c) Express $P \rightarrow (P \rightarrow Q)$ using $\downarrow$ only. 2

(d) Prove by contradiction method that the following premises are consistent:

$(r \rightarrow \bar{q}, r V S, S \rightarrow \bar{q}, P \rightarrow q) \rightarrow \bar{p}$. 4

8. (a) Let $f(n)$ and $g(n)$ be asymptotically increasing non-negative functions. Prove that :
$\max(\text{fen}), g(n)) = \Theta(f(n) + g(n))$ 3

(b) Use Master Theorem to find the tight upper bound for the given recurrence relation :
$T(n) = 3T\left(\frac{n}{4}\right) + n^2$ 3

(c) Is it true that $x^3 = \Theta(7x^2)$?
Justify your answer using the basic definition of the "theta" notation. 4

(d) Express the sum $\sum_{k=1}^{n} (2k-1)$ as a function of n. 2

(e) Use the substitution method to prove that for the recurrence relation : $T(n) = 3T\left(\frac{n}{3}\right) + n$, the solution is given by O (n lgn). 4