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1790

Your Roll No.

B.Sc. (Hons.) Computer Sc. / III Sem. A

Paper 303 – Algebra

(Admissions of 2001 and onwards)

Time : 3 Hours

Maximum Marks : 75

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*Attempt all questions. All questions carry equal
marks. Use of scientific calculator is permitted.*

1. Show that the set $C = \{1, w, w^2\}$ where w is an imaginary cube root of unity, is a group with respect to multiplication.
2. Show that the mapping $\phi (a + ib) = a - ib$ is an isomorphism of the group of complex numbers \mathbb{C} under addition. Show that ϕ preserves complex multiplication as well – that is $\phi (xy) = \phi(x) \phi(y)$ for all x and y in \mathbb{C} .

[P.T.O.]

3. Let f be a morphism from ring to ring R' . Prove that $\ker f$ is an additive subgroup of R . Also show that if $a \in \ker f$ then $ra \in \ker f$ for all $r \in R$.

4. Let A be a linear transformation whose matrix is $\begin{pmatrix} 3 & 0 \\ 4 & 2 \end{pmatrix}$. Find a linear transformation D with diagonal matrix and find a linear transformation P such that $A = PDP^{-1}$. Also calculate $\begin{pmatrix} 3 & 0 \\ 4 & 2 \end{pmatrix}^6$.

5. Show that the following set of elements in \mathbb{R}^3 form subspace,

$$\{(x, y, z) \mid x = y \text{ and } 2y = z\}.$$

Give an example which shows that union of two subspaces may not be a subspace.

6. Define a convex set of a vector space. If S be a convex set in vector space V and $w \in V$, show that.

$$w + S = \{w + v \mid v \in S\}$$

is a convex set.

7. Let A be a set of books. Let R_1 be a binary relation on A such that (a, b) is in R_1 if book 'a' costs more and contains fewer pages than book 'b'. In general, is R_1 reflexive ? Symmetric ? Antisymmetric ? Transitive ?
8. Let $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear mapping for which $F(1, 2) = (2, 3)$ and $F(0, 1) = (1, 4)$, Find a formula for $F(a, b)$.
9. Let $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the mapping defined by $F(x, y) = (x - y, x - 2y)$. Is F invertible ? If yes, find F^{-1} .
10. Let V be the space generated by functions $f_1(t) = \sin t$, $f_2(t) = \cos t$, let $D : V \rightarrow V$ be the mapping such that $D(f) = \frac{d^2 f}{dt^2}$. Show that D is linear and describe kernel of D .
11. Find the dimension of the set of solutions of the following system of equations and determine this set in \mathbb{R}^3

$$2x + y + z = 1$$

$$y - z = 0.$$

12. State and prove Bessel inequality.
13. Let δ consists of the following vectors in \mathbb{R}^4 :
 $u_1 = (1, 1, 0, -1)$, $u_2 = (1, 2, 1, 3)$, $u_3 = (1, 1, -9, 2)$,
 $u_4 = (16, -13, 1, 3)$
Show that δ is an orthogonal basis of \mathbb{R}^4 . What is the dimension of S^\perp ?
14. Find the area of the parallelogram such that three corners of the parallelogram P are given by $(-3, 2)$, $(1, 4)$ and $(-2, -7)$.
Let $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear map defined by
 $L(x, y) = (x + 2y, x - y)$.
Find the area of $L(P)$.
15. Find the maximum and minimum of the function
 $f(x, y) = 3x^2 + 5xy - 4y^2$
on the unit circle.