1796

Your Roll No.............................

A

B.Sc. (H) / Computer Science / IV Sem.

Paper 404 - DIFFERENTIAL EQUATIONS

(Admissions of of 2001 and onwards)

Time : 3 hours

Maximum Marks :75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions.
All questions carry equal marks.
Non - Programmable calculator is allowed.

1. The curve y(x) of an inextensible flexible homogeneous cable hanging between two fixed points is obtained by solving $y'' = K\sqrt{1+y'^2}$ where the constant K depends on the weight. Find and graph y(x) assuming that K = 1 and those fixed points are (-1, 0) and (1, 0) in the vertical xy - plane.

2. Show that $y = x e^{5x}$ is a solution of the differential equation

   $y'' - 10y' + 25y = 0$

   Find a linearly independent solution by reducing the order.

3. Define Euler - Cauchy equation. Convert the differential equation $4x^2y'' + 24xy' + 25y = 0$ into the differential equation with constant coefficients and solve.

4. Define the Wromkian of two solutions $y_1(x)$ and $y_2(x)$ of the second order differential equation. State a necessary and sufficient condition for the solutions $y_1(x)$ and $y_2(x)$ to be linearly independent. Prove that $e^x \cos x$ and $e^x \sin x$ are linearly independent solutions of the differential equation.

   $y'' - 2y' + 2y = 0.$

5. Use the method of variation of parameters to find a general solution of the differential equation.

   $xy'' - y' = (3 + x)x^2 e^x$

6. Find the particular integral and hence find the general solution of the differential equation.

   $y'' - 2y' + 4y = e^x \cos x + x \sinh x$

7. State the condition for the existence of power series solution of the 2nd order linear differential equation. Does the power series solution of the differential equation.

   $y' = \dfrac{y}{x} + 1$ exists ? Discuss.

8. Define Bessel's function of first kind of order n

   show that $\int x^{-\nu} J_{\nu+1}(x)\,dx = -x^{-\nu} J_{\nu}(x) + C$

   and deduce $\int x^3 J_0(x)\,dx$

9. Show the following

(i) $J_{\nu-1}(x) - J_{\nu+1}(x) = 2J'_{\nu}(x)$

(ii) $J_{-1/2}(x) = \sqrt{\dfrac{2}{\pi x}}\cos x$

10. Determine the radius of convergence of the following power series :

(i) $\sum\limits_{m=0}^{\infty} m^m (x-3)^m$

(ii) $\sum\limits_{m=0}^{\infty} \dfrac{1}{4^m m^m} x^{2m}$

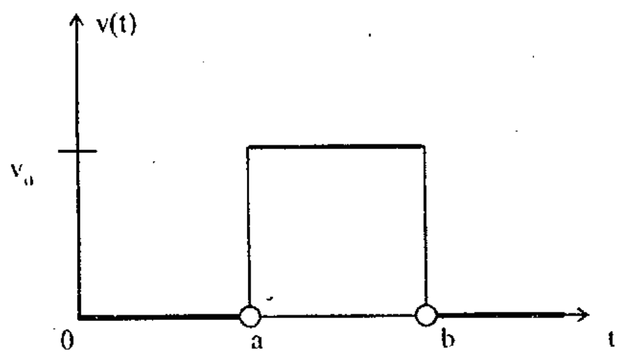
11. Using Laplace transform, solve the integral equation $y(t) = e^t + \int_0^t y(\tau)\,\mathrm{Sin}\,2(t-\tau)\,d\tau$.

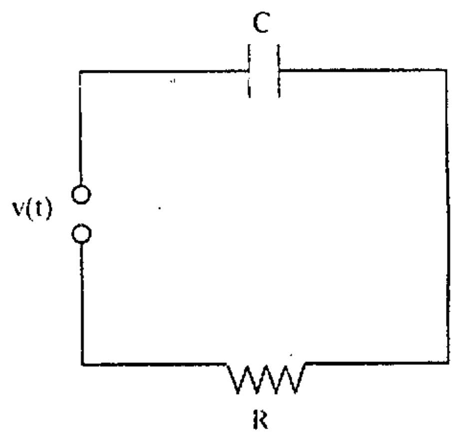
12. Find the inverse laplace transform of the f.

(i) $\dfrac{s^2+1}{s^3-2s^2-8s}$

(ii) $ln\left(\dfrac{s^2+1}{s^2+4}\right)$

13. Find the current $i(t)$ in the circuit in the figure given below if a single square wave with voltage V_0 is applied. The circuit is assumed to be quiescent before the square wave is applied.





Figure

14. Use Laplace transform to solve the initial value problem

$$x'' + 4x = f(t)$$
$$x(0) = x'(0) = 0$$

where
$$f(t) = \begin{cases} \cos 2t, & 0 \le t \le 2\pi \\ 0, & t > 2\pi \end{cases}$$

15. Solve the system of differential equation using laplace transform.

$$y_1'' + y_2 = -5 \cos 2t$$

$$y_2'' + y_1 = 5 \cos 2t$$

$$y_1(0) = 1, \; y_1'(0) = 1$$

$$y_2(0) = -1, \; y_2'(0) = 1$$