

[This question paper contains 6 printed pages.]

911

Your Roll No.

B.Sc. (Hons.) / II

C

CHEMISTRY – Paper X

(Mathematics – II)

Time : 3 Hours

Maximum Marks : 55

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

Question 1 is compulsory.

*Attempt 2 questions from Section A
and three questions from Section B.*

1. (a) Find the rank of the matrix

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \quad (4)$$

- (b) Find the Characteristic roots of the matrix

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \quad (3)$$

P.T.O.

- (c) For what values of λ, μ the system of simultaneous equations

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

have a unique solution. (3)

SECTION A

2. (a) Show that the sequence $\langle a_n \rangle$ defined by

$$a_1 = (2)^{1/2}, a_{n-1} = (2a_n)^{1/2} \text{ for every } n \geq 2$$

is convergent. Also find its limit. (3)

- (b) Give an example each of

(i) Bounded sequence which is not convergent

(ii) Divergent sequence

(iii) Neither bounded nor monotonic (3)

- (c) Test for convergence the series

$$1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots \quad (3)$$

3. (a) Test for convergence the series

$$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \quad (3)$$

(b) Test for convergence the series

$$\sum_{n=1}^{\infty} \left(\sqrt{n^4 + 1} - \sqrt{n^4 - 1} \right) \quad (3)$$

(c) Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1.2.3 \dots (2n-1)}{2.4.6 \dots 2n} x^n \quad (x > 0) \quad (3)$$

4. (a) Prove that

$$\beta(m, n) = \frac{\sqrt[m]{n}}{\sqrt[m+n]{n}} \quad (m > 0, n > 0) \quad (3)$$

(b) Evaluate $\int_0^1 x^5 (1-x)^{4/3} dx$ (3)

(c) Prove that

$$\beta(m, n) \beta(m+1/2, n+1/2) = \frac{\pi m^{-1}}{2^{4m-1}} \quad (3)$$

5. (a) Evaluate

$$\iiint (x + y + z) dx dy dz$$

over the tetrahedron $x = 0$, $y = 0$, $z = 0$ and
 $x + y + z = 1$. (4½)

- (b) Evaluate

$$\iint x^2 y^2 dx dy$$

over the positive quadrant of the circle $x^2 + y^2 = 1$.
(4½)

SECTION B

6. (a) Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys and 1 girl and 3 boys. One child is selected at random from each group. Find the probability that the three selected consist of 1 girl and 2 boys. (3)
- (b) If A and B are disjoint events. $P(A) = 0.5$, $P(A \cup B) = 0.6$. Find $P(B)$. (3)
- (c) Out of 11 tickets consecutively numbered, three are drawn at random. Find the probability that the numbers on them are in A.P. (3)
7. (a) If the sum and product of the mean and variance of a Binomial distribution are 24 and 128. Find the distribution. (3)

- (b) Show that a Poisson distribution with unit mean, mean deviation about mean is $2/e$. (3)
- (c) What is the expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability p of success in a trial. (3)
8. (a) Show that for a normal distribution mean, median and mode coincide. (4½)
- (b) If X_1 and X_2 are random variables, show that
- $$\text{Var}(X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2) + \text{Cov}(X_1, X_2)$$
- (4½)
9. (a) For a bivariate distribution the lines of regression are $3x + 12y = 19$ and $3y + 9x = 46$. Find the mean of the distribution and the correlation coefficient. (4½)
- (b) Fit a straight line to the following data :
- | | | | | | | |
|-----|----|----|----|----|----|----|
| x : | 0 | 5 | 10 | 15 | 20 | 25 |
| y : | 12 | 15 | 17 | 22 | 24 | 30 |
- (4½)
10. (a) A biased coin was thrown 400 times and head resulted 240 times. Find the standard error of the observed proportion of the heads and find the limits of probability of getting head in a throw of the coin. (4½)

(b) A dice is thrown 264 times with following results :

No. on Dice :	1	2	3	4	5	6
Frequency :	40	32	28	50	54	60

test the hypothesis that the dice is biased.

(4½)

[χ^2 for 5 d.f. at 5% level = 11.07]