[This question paper contains 4 printed pages.]

904 *Your Roll No.* .....

B.Sc. (Hons.) / I **C**

CHEMISTRY - Paper IV

(Mathematics - I)

Time : 3 hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** *questions in all, selecting at least* **one** *question from each Section.*

SECTION A

1. (a) If $y = \sin\left(m \sin^{-1} x\right)$, show that

$$(1 - x^2)y_{n+2} = (2n + 1)xy_{n+1} + (n^2 - m^2)y_n \qquad (3)$$

(b) Obtain Maclaurin series expansion of $\log(1+x)$. (3)

(c) If $u = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$ show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \qquad (3)$$

2. (a) Verify Lagrange's mean value theorem for the function

$f(x) = (x-1)(x-2)(x-3)$ in $[1,4]$. (3)

P.T.O.

(b) Show that the function $3x^3 - 9x^2 + 9x + 7$ is strictly increasing in every interval. (3)

(c) Evaluate $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{1/x^2}$. (3)

3. (a) Show that the tangent and the normal at any point of the curve $x = a e^{\theta}(\sin\theta - \cos\theta)$ and $y = a e^{\theta}(\sin\theta + \cos\theta)$ are equidistant from the origin. (3)

(b) Find the asymptotes of the curve $x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$. (3)

(c) Trace the curve : $x(x^2 + y^2) = a(x^2 - y^2)$. (3)

SECTION B

4. (a) Find the area between the curve $y^2(2a - x) = x^3$. (5)

(b) Show that the length of the loop of the curve $3ay^2 = x(x - a)^2$ is $\frac{4a}{\sqrt{3}}$. (4)

5. Evaluate any two the following integrals :

(a) $\int_0^{\pi} \frac{\cos x}{1 + \sin^2 x} dx$

(b) $\int \frac{x dx}{(x + 2)\sqrt{x + 1}}$

(c) $\int \sqrt{\frac{x+2}{2x+3}} \frac{1}{x} dx$ (9)

6. Solve any two of the following differential equations :

(a) $(x + 2y^3) dy = y dx$

(b) $(y - 2x) dy = (x - 2y) dx$

(c) $(D^2 + 4) y = e^{-2x} - \sin 3x$

(d) $(x^2D^2 - 2xD + 2) y = x^3$ (9)

SECTION C

7. (a) Find the value of λ so that the equation $2x^2 + xy - y^2 - 11x - 5y + \lambda = 0$ may represent a pair of the straight lines. (5)

(b) Find the equation of the circle whose diameter is the common chord of the circles

$x^2 + y^2 + 2x + 3y + 1 = 0$, $x^2 + y^2 + 4x + 3y + 2 = 0$. (4)

8. (a) Prove that the locus of the mid points of all chords of the parabola $y^2 = 4ax$ drawn through the vertex is the parabola $y^2 = 2ax$. (4)

(b) Show that the polar of any point on the ellipse w. r. t the hyperbola touches the given ellipse. (5)

P.T.O.

9. (a) Find the equation of the plane through the intersection of the planes
$x - 2y + 3z + 4 = 0$ and $4x + 3y + 2z + 7 = 0$
and passes through point P(1,1,1). (4)

(b) Find the equation of the sphere which touches the sphere
$x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$ at the point (1,1,-1) and passes through origin. (5)

SECTION D

10. (a) Solve : $x^4 - x^3 + x^2 - x + 1 = 0$. (3)

(b) Prove that $16 \sin^5\theta = \sin 5\theta - 5 \sin 3\theta + 10 \sin\theta$. (3)

(c) Prove that $\left(\sqrt{3} + i\right)^n + \left(\sqrt{3} - i\right)^n = 2^{n+1} \cos\frac{n\pi}{6}$. (3)

11. (a) Solve $x^3 - 3x^2 + 4 = 0$ given that two of its roots being equal. (3)

(b) If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$. Find the value of
$\Sigma\alpha^2\beta$ (3)

(c) If α, β, γ are the roots of the equation $x^3 - 3x^2 + 6x - 2 = 0$, form the equation whose roots are $\beta^2 + \gamma^2$, $\alpha^2 + \gamma^2$, $\alpha^2 + \beta^2$. (3)