Your Roll No. ....

1984

B.Sc. (Hons.)/(Computer Science)/V Sem. C

Paper 504 : NUMERICAL ANALYSIS AND

SCIENTIFIC COMPUTING

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions in order.

Parts of a question must be answered together.

Use of non-programmable scientific calculator is allowed.

1. (*a*) Define order of convergence for a sequence. Prove that the sequence of iterates (X_n) to find reciprocal of a positive real number N developed by using Newton's method converges to the actual value $\frac{1}{N}$ if and only if initial approximation X_0 satisfies the inequality $0 < x_0 < \frac{2}{N}$. 5

(b) Write an algorithm for bisection method to find an approximate root of an equation $f(x) = 0$. Apply it to find a smallest positive root of $x - \tan(x) = 0$ correct upto 3 places of decimal. 5

2. (a) Prove that there exists a unique polynomial of degree at most n that interpolates y_i at x_i, $i = 0, 1, \ldots\ldots, n$, where $x_0, x_1, \ldots\ldots, x_n$ are all distinct. 5

(b) Use forward difference interpolation formula to find $f(2.15)$ for the following data : 5

x_i	$f(x_i)$
2.0	1.414214
2.1	1.449138
2.2	1.483240
2.3	1.516575
2.4	1.549193

(c) Derive interpolation error formula in the form :

$$f(t) - p_n(t) = (t - x_0)(t - x_1) \ldots\ldots\ldots (t - x_n)$$

$$f[x_0, \ldots\ldots, x_n, t]$$

where $p_n(t)$ denotes interpolating polynomial of degree at most n that interpolates $f(x_0), f(x_1), \ldots\ldots f(x_n)$ at distinct points $x_0, x_1, \ldots\ldots, x_n$. 5

3. (a) Let f be a function having continuous derivatives upto order 2. Derive composite trapezoidal integration error formula for f on $[a, b]$ in the form :

$$E_n(f) = \frac{-(b-a)h^2}{12} f''(n)$$

where $n \geq 1$ and $h = \frac{b-a}{n}$. 5

(*b*) Derive Gaussian quadrature formula for $\int_{-1}^{1} f(x)\,dx$ with two node points and use it to find approximate value of :

$$\int_0^{\pi} e^x \cos(x)\,dx .$$ 5

4. (*a*) Derive an error formula for $f''(x)$ by using the method of undetermined coefficients. 5

(*b*) Use modified Euler's method to find solution to :

$$\frac{dy}{dx} = y^2 + x^2,\ y(1) = 0,$$

at $x = 2$ and using $h = 0.2$. 5

(*c*) Use fourth order Runge-Kutta method with $h = 0.2$ to find $y(1)$ for the following differential equation :

$$\frac{dy}{dx} = \frac{1}{(x+y)},\ y(0) = 2 .$$ 5

5. (*a*) Use Gauss elimination method to solve the following system of equations : 5

$$4x - 2y + z = 15$$

$$-3x - y + 4z = 8$$

$$x - y + 3z = 13.$$

(*b*) Evaluate 1-norm, 2-norm and Frobenius norm of the matrix : 5

$$\begin{bmatrix} 5 & -9 & 6 \\ 2 & -7 & 4 \\ 1 & 5 & 8 \end{bmatrix}.$$

6. (*a*) Use Gauss-Seidel method to solve : 5

$$x - 2y + z = 0$$

$$3x - y + 4z = 6$$

$$x + y + 3z = 5.$$

(*b*) Use Rayleigh-Ritz method to solve $y'' + y = 3x^2$ with boundary points (0, 0) and (2, 3, 5). 5

(*c*) Consider a least square linear approximation to the function $y = e^x$ on [0, 1]. 5