This question paper contains 4+2 printed pages]

Your Roll No...............

1951

B.Sc. (Hons.) Computer Science/IV Sem. C

Paper 404—Differential Equations

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions.

All questions carry equal marks.

Non-programmable calculator is allowed.

1. A certain culture of bacteia grows at a rate that is proportional to the number present. It is found that the number doubles in 4 hours, how many bacteria may be expected at the end of 12 hours.

2. Consider the following differential equation :

$$2x^2y'' - xy' - 2y = 0.$$

Use method of reduction of order to find the basis of solution for it.

3. Define Euler-Cauchy equation. Convert the differential equation :

$$x^2y'' + xy' + 9y = 0$$

into differential equation of constant coefficients and solve it.

4. Are the following functions linearly independent or dependent on the given interval ?

(*i*) $e^x, e^{|x|}$ for $x \in \mathbf{R}$;

(*ii*) $\cos 2x, \cos |x|$, $0 < x < 2\pi$.

5. Use method of variation of parameters to find general solution of differential equation :

$$x^2y'' + xy' + 9y = 3e^{2x}.$$

6. Find the particular integral and hence find the general solution of differential equation :

$$(D^3 - 7D - 6)y = e^{2x}(1 + x).$$

7. Solve the differential equation :

$$y' = \frac{y}{x} + 1$$

for y as a power series at $x_0 = 1$. Why this cannot be solved for a power series at $x_0 = 0$?

8. Define Bessel's function of first kind of order n.

Show that :

$$\int x^{-\gamma} J_{\gamma+1}(x)\,dx = x^{-\gamma} J_{\gamma}(x) + c$$

and deduce

$$\int x^3 J_0(x)\,dx.$$

9. Show the following :

(*i*) $J_{\nu-1}(x) + J_{\nu+1}(x) = \frac{2\nu}{x} J_\nu(x)$;

(*ii*) $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

10. Find the radius of convergence of power series :

(*i*) $\sum_{m=1}^{\infty} \frac{3^{2m}}{m}(x-1)^m$;

(*ii*) $\sum_{m=0}^{\infty} m^m x^m$.

11. Solve the integral equation :

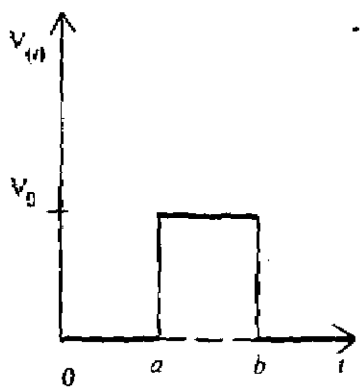
$$y(t) = t + \int_0^t y(\tau) \sin(t-\tau) d\tau.$$

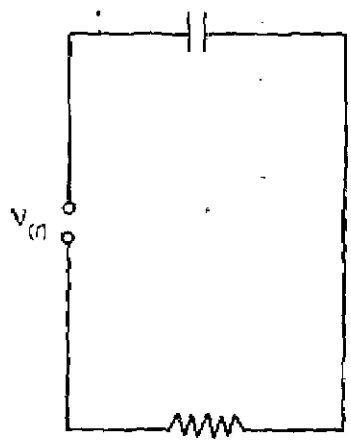
12. Find the inverse Laplace transform of the following :

(*i*) $\frac{s}{\left(s+\frac{1}{2}\right)^2 + 1}$;

(*ii*) $\log\left(\frac{s-2}{s+2}\right)$.

13. Find the current $i(t)$ in the circuit in the figure given below if a single square wave with voltage V_0 is applied. The circuit is assumed to be quiescent before the square wave is applied.





14. Solve the given initial value problem using Laplace transform :

$$y'' + y' - 6y = 0 \qquad y(0) = 2,\ y'(0) = -1.$$

15. Solve the given system of differential equation by means of Laplace transforms :

$$y''_1 + y_2 = -5 \cos 2t$$

$$y''_2 + y_1 = 5 \cos 2t$$

$$y_1(0) = 1, \quad y'_1(0) = 1, y_2(0) = -1, \quad y'_2(0) = 1.$$