This question paper contains 4+2 printed pages]

Your Roll No................

1978

B.Sc. (Hons.) Computer Sc. /III Sem. C

Paper 303—Algebra

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all the *seven* questions from Section A and any *four* questions from Section B.

Section A

1. Define :

(*i*) A monoid

(*ii*) A normal group

(*iii*) A ring

Illustrate these definitions with an example each. 5

P.T.O.

2. If A = {1, 2, 3, 4} and B = {*a*, *b*, *c*} what is :

(*i*) A × B

(*ii*) Is A poset under a partial order "IS LESS THAN".

(*iii*) Is this poset in (*ii*) a chain ? (Under "IS LESS THAN")

(*iv*) Is the set A a group with respect to binary composition * as ordinary multiplication, jusitfy ?

(*v*) Is the set A a ring with respect to two binary compositions +, . as the ordinary addition and multiplication of real nos ? Justify. 5

3. Show that the set of all elements (x, y, z) in $\mathbf{R}^3$ such that $x + y = 3z$ forms a sub-space of $\mathbf{R}^3$. 5

4. (*i*) Determine whether the matrix $A = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$ is positive definite.

(*ii*) Show that for the above $A = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$ trace $(A^2) \geq 0$ equality $= 0$ holds only when each entry of A is zero. 5

5. Let $\alpha_1, \alpha_2, \ldots\ldots\ldots, \alpha_n \in \mathbf{R}$ be distinct real numbers $\neq 0$. Show that the functions $e^{\alpha_1 t}$ .... $e^{\alpha_n t}$ are linearly independent over the field of real numbers **R**. 5

6. What is the dimension of (the vector space of) : 5

(*i*) Upper triangular $n \times n$ matrices over **R**.

(*ii*) Symmetric 3×3 matrices over **R**.

7. Find the eigen values and corresponding eigen vectors of the matrix $\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$. 5.

Section B

8. (*a*) Define a parallelogram spanned by two distinct vectors v_1 and v_2 of a vector space over reals. Draw the parallelogram spanned by the vectors (2, −1) and (1, 3) in $\mathbf{R}^2(\mathbf{R})$. 5

(*b*) Show that the set of all $(x, y) \in \mathbf{R}^2(\mathbf{R})$ such that $x = y$ forms a subspace of $\mathbf{R}^2(\mathbf{R})$. 5

9. (*a*) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 7 \\ 2 & 4 & -1 \end{pmatrix}$. 5

(*b*) Find the rank of the matrix $\begin{pmatrix} -1 & 0 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 7 \end{pmatrix}$. 5

10. (*a*) Let V be the subspace of functions generated by two functions $f(t) = t$ and $g(t) = t^2$. Find an orthonormal basis for V. 5

(*b*) Show that the vectors (1, 1, 1) and (0, 1, –2) are linearly independent over reals **R**. 5

11. (*a*) Express the vector (4, 3) in terms of vectors (2, 1) and (-1, 0) as a linear combination. Are the vectors (2, 1) and (–1, 0) lindependent over **R** ? 5

(*b*) Show that SL(2, **R**) is a normal subgroup of the group GL(2, **R**). 5

12. (*a*) If H and K are two subgroups of a group G, then $H \cap K$ is also a subgroup of G. 5

(*b*) Show that the set $P_2(x)$ of all real polynomials in x of degree at most 2 is a vectorspace over reals. 5

13. (*a*) If $\theta \in \mathbf{R}$, then the matrix $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ always has an eigen vector in $\mathbf{R}^2$. 5

(*b*) Is the map $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined by $T(x, y, z) = (x, z)$ linear ? Find the image of $(1, 0, -1) \in \mathbf{R}^3$. 5