This question paper contains 4+1 printed pages]

Your Roll No. ....

1972

B.Sc. (Hons.)/Computer Sc./I Sem. **C**

Paper 103—CALCULUS–I

(Admissions of 2001 to 2010)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *ten* questions.

Each question carries 7½ marks.

1. It can be shown that the inequalities :

$$\frac{1}{2} - \frac{x^2}{24} < \frac{1 - \cos x}{x^2} < \frac{1}{2}$$

hold for values of x close to zero. What, if anything, does this tell you about :

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} ?$$

Give reasons for your answer.

P.T.O.

2. The approximation :

$$\sqrt{1+x} = 1 + \frac{x}{2}$$

is used when x is small. Estimate the error when $|x| < 0.01$.

3. Find the values of x for which the geometric series :

$$\sum_{n=0}^{\infty} (-1)^n (x+1)^n$$

converges. Also, find the sum of the series for those values of x.

4. Define an absolutely convergent series. Test the following series for absolute convergence :

$$\sum_{n=1}^{\infty} (-1)^n \frac{(n+1)^n}{(2n)^n}.$$

5. For the function $f(x) = \sqrt{x+1}$, $x_0 = 0$ and $\in = 0.1$, find L and a number $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$, the inequality $|f(x) - L| < \in$ holds.

6. Show that the function :

$$f(x) = 2x^4 - x - 1$$

has exactly one zero in the interval [–1, 0].

7. Graph the function :

$$f(x) = |x^2 - 1|.$$

Include the coordinates of local extreme points and inflexion points if any.

8. Evaluate :

(*i*) $$\lim_{x \to 0} \left(\frac{e^x - 1 - x - \frac{x^2}{2}}{x^3} \right)$$

(*ii*) $$\lim_{x \to 2} \left(\frac{x^5 - 32}{x^3 - 8} \right).$$

9. Show that a closed cylindrical can of fixed volume has minimum total surface area if its height and diameter are equal.

10. Given :

$$\frac{d\vec{r}}{dt} = 2t\hat{i} + \frac{1}{(t+1)^2}\hat{j} - e^t\hat{k}$$

and $\vec{r}(0) = \hat{i}$,

find $\vec{r}(t)$.

11. Show that the function :

$$f(x, y) = \frac{2x^2 y}{x^4 + y^2}$$

has no limit as (x, y) approaches $(0, 0)$.

12. Find $\frac{\partial w}{\partial x}$ at the point $(x, y, z) = (2, -1, 1)$ if

$$w = x^2 + y^2 + z^2,$$

$$z^3 - xy + yz + y^3 = 1 \quad .$$

and x and y are the independent variables.

13. Find all the local maxima, local minima, and saddle points of the function :

$$f(x, y) = x^2 + 3xy + 3y^2 - 6x + 3y - 6.$$

14. Find the point $P(x, y, z)$ closest to the origin on the plane

$2x + y \quad z \quad 5 - 0$.

15. Find a quadratic approximation of $f(x, y) = \sin x \sin y$ at the origin. Estimate the error in the approximation if $|x| \leq 0.1$ and $|y| \leq 0.1$.

Or

Assuming the validity of differentiation under the integral sign, show that :

$$\int_0^{\pi/2} \frac{\log(1 + \cos\beta \cos y)}{\cos y} \, dy = \frac{\pi^2 - 4\beta^2}{8}.$$