[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5005 **D** **Your Roll No...............**

Unique Paper Code : 235166

Name of the Course : **B.Sc. (Hons.) Computer Science, B.Sc. (Mathematical Science), B.Sc. (Physical Sciences)**

Name of the Paper : Calculus and Matrices (MAPT 101)

Semester : I

Time : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** questions from each section.

SECTION I

1. (a) Is the following set of vectors a basis for R^2

$$\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(b) Solve the system of equations :

$$x + y + z = 7$$

$$x + 2y + 3z = 16$$

$$x + 3y + 4z = 22$$

(c) Examine whether the set $V = \{(a,b^2) : a, b \in R]$ is a subspace of R^2. If yes, give its geometrical interpretation. (4,4,4)

2. (a) Is the transformation $T : R^3 \to R^3$; $T(x,y,z) = (x + y, x + z, x)$ linear. Justify.

P.T.O.

(b) Find the characteristic equation, eigenvalues and eigen vector corresponding to any one eigen value for the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$$

(c) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ using elementary operations.

(4,4,4)

3. (a) Find the image of the unit square with vertices (0,0), (0,1) (1,1), (1,0) under a translation by vector (1,1).

(b) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 3 & 0 & 1 \end{bmatrix}$.

(c) Find the values of c for which the set of vectors $\{(2, -c), (2c + 6, 4c)\}$ in R^2 are linearly dependent. (4,4,4)

SECTION II

4. (a) Sketch the graph of $y = |x - 3| + 7$. Mention the transformation used at each step.

(b) A bacteria culture is known to grow at a rate proportional to the number present. After one hour, 1000 bacteria are observed in the culture and after 4 hours, it is 3000. Determine the number of bacteria originally present in the culture.

(c) Draw the level curve of $f(x,y) = 9x^2 + 25y^2$ of height k = 1, 2. (6,6,6)

5. (a) If $y = e^{m \sin^{-1}(x)}$, show that

$$\left(1 - x^2\right)y_{n+2} - (2n + 1)xy_{n+1} - \left(n^2 + m^2\right)y_n = 0$$

(b) Show that $z = e^{-y} \cos(x)$ is a solution of Laplace's equation.

(c) Find Taylor series generated by $f(x) = \cos(2x)$ about $x = 0$ (assuming the possibility of its expansion). (6,6,6)

6. (a) Discuss the convergence of the sequences :

(i) $\left\langle \frac{2n-1}{3n} \right\rangle$

(ii) $\left\langle \frac{\cos^2(n)}{n^2} \right\rangle$

(b) If $z = 3xy - y^3 + (y^2 - 2x)^{3/2}$. Show that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}.$$

(c) Find $\frac{d^n y}{dx^n}$ where $y = \cos^3 x$. (6,6,6)

SECTION III

7. (a) Simplify $\left(\frac{1 + \cos\theta + i\sin\theta}{1 + \cos\theta - i\sin\theta}\right)^n$.

(b) Find the equation of the circle described on the join of the points $1 + i$ and $2 - i$ as extremities of one of its diameters. (4,3.5)

8. (a) Use De Moivre's Theorem to solve the following equation :

$$z^7 + z = 0$$

(b) Find the equation of the straight line joining the points whose affixes are $1 - i$ and $2 - 5i$. (4,3.5)

9. (a) Form an equation of lowest degree with real coefficients that has $2 + 3i$ and $3 - 2i$ as two of its roots.

(b) Find all the values of $\left(\sqrt{3} - i\right)^{2/5}$. (4,3.5)

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