[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 6227 **F-5** **Your Roll No.**4101439052

Unique Paper Code : 2221501

Name of the Paper : Quantum Mechanics and its Applications I

Name of the Course : **Erstwhile FYUP B.Sc. (Hons.) Physics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question No. **1** is compulsory.
4. **All** questions carry equal marks.

1. Attempt any **five** questions of the following:

(a) Discuss the various postulates of Quantum Mechanics.

(b) Using the operator representation of p_x, p_y and p_z

$$[x, p_y] = 0,$$

$$[p_x, p_y] = 0,$$

$$[y, p_y] = i\hbar$$

(c) Write the general solution of the time dependent Schrodinger equation with time-independent potential.

(d) Estimate the ground state energy of a particle in an one dimensional box of L using uncertainly relation.

(e) Obtain the expression of effective mass of an electron in a metal and use the energy spectrum of electron to discuss whether the effective mass of an electron can be negative.

(f) Write the expression for ground state energy of a quantum harmonic oscillator and discuss its physical significance.

(g) What is the expectation value of K.E of electron in a hydrogen atom if the ground state wave function of electron in the atom is $Q_{100} = \frac{1}{\sqrt{\pi}} \frac{e^{-r/a_o}}{a_o^{3/2}}$

(h) Prove that $\sigma \times \sigma = 2i\,\sigma$ (3×5=15)

2. (a) For the Gaussian wave packet given by

$$(x,0) = \frac{1}{(\pi\sigma_o^2)^{1/4}} e^{-x^2/2\sigma_o^2} e^{ip_o x}/\hbar$$

which describes a particle localized within a distance σ_o moving with an average momentum P_o with a momentum spread approximately equal to $\hbar/\sigma_o$.

Evaluate $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$, $\langle p^2\rangle$ and show that $\Delta x\, \Delta p = \frac{\hbar}{2}$. (10)

(b) Using the time dependent Schrodinger equation in one dimension, discuss the concept of stationary states. (5)

3. (a) Discuss Kronig-Penney model and solve the Schrodinger equation for the motion of electrons in a one dimensional periodic potential leading to energy band structure in a solid. (10)

(b) Using the idea of energy bands, briefly explain the difference between a conductor, a semiconductor and an insulator. (5)

4. (a) Solve the Schrodinger equation for a particle having energy E in a square well potential defined by

$V(x) = V_0$ for $x < -a$

$= 0$ for $-a < x < a$

$= V_0$ for $x > a$

where $E < V_0$ (10)

(b) Discuss the significance of the quantum numbers n, l, m_l and m_s. (5)

5. (a) Set up time independent Schrodinger equation for a particle of mass m performing simple harmonic motion of frequency w. Show that the allowed energy must be of the form

$$E = \left(n + \frac{1}{2}\right)h\omega$$ where n is an integer. (10)

(b) Sketch the wave function and corresponding probability density for the simple harmonic oscillator for n = 1 and 2 states. (5)

6. (a) Write the Schrodinger equation for hydrogen atom in spherical polar coordinates. Split the equation into three equations, separately depending on r, θ, φ dependent wave functions. Obtain the solution of the radial equation. (10)

(b) The wave function for hydrogen atom in 1s state is

$$R_{1s}(r) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_o}\right)^{3/2} e^{-r/a_o}$$

where a_0 = Bohr radius.

Calculate the expectation value of potential energy of the electron in this state. (5)

7. (a) Describe Stern-Gerlach experiment. Discuss the significance of this experiment. Why is an inhomogeneous magnetic field required? (10)

(b) A beam of silver atom with a velocity of 10^6 cm/s passes through a magnetic field of gradient 50 W/m^2/cm for a distance of 10 cm. What is the separation between the two components of the beam as it comes out of the magnetic field? (5)

Physical constants:

$h = 6.6 \times 10^{-34}$Js;

$e = 1.6 \times 10^{-19}$ C

$m = 3 \times 10^{-31}$ kg