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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8802**

Unique Paper Code : **235103** **C**

Name of the Paper : **I.2 (Analysis-I) (Admissions of 2011 and onwards)**

Name of the Course : **B.Sc. (Hons.)/Maths. Part I**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

1. (*a*) State and prove triangle inequality and show that : 5

$$||a| - |b|| \leq |a - b|, \ \forall \ a, b \in \mathbb{R}.$$

(*b*) Define bounded above and bounded below sets. Give examples of sets which are :

(*i*) Bounded above but not bounded below.

(*ii*) Bounded below but not bounded above.

(*iii*) Neither bounded above nor bounded below. 5

(*c*) If x and y are two real numbers with $0 < x < y$, then there exists an irrational number z such that $x < z < y$. 5

2. (*a*) (*i*) Given $S = \left\{1 + \frac{1}{n} : n \in \mathbb{N}\right\}$, show that $\inf S = 1$. 2½

(*ii*) Given $A = \left\{1 - \frac{1}{n} : n \in \mathbb{N}\right\}$, show that $\sup A = 1$. 2½

P.T.O.

(*b*) Suppose A and B are two non-empty bounded subsets of R, and

$$A + B = \{a + b : a \in A, b \in B\}.$$

Prove that :

$$\sup (A + B) = \sup A + \sup B.$$ 5

(*c*) Define cluster point of a set of real numbers and show that $A = \left\{\frac{1}{n} : n \in N\right\}$ is not a closed set but $A \cup \{0\}$ is closed. 5

3. (*a*) (*i*) Write x_5, x_{11}, x_{16} and x_{50} for the sequence (x_n) where : 2

$$x_n = 3 + 2(-1)^n, n \in N.$$

(*ii*) Prove that :

$\lim\left(\frac{2n}{2n+1}\right) = 1$ by using the definition of limit of a sequence. 3

(*iii*) Show that $\lim\left(\frac{\sin n}{n}\right) = 0$. 2½

(*b*) (*i*) Determine limit of sequence $\left(\left(1 + \frac{1}{2n}\right)^{3n}\right)$. 2½

(*ii*) If $0 < b < 1$, then show that $\lim (b^n) = 0$. Further determine

$\lim\left(\frac{a^{n+1} + b^{n+1}}{a^n + b^n}\right)$ for $0 < a < b$. 5

(*c*) (*i*) Show that every convergent sequence is bounded. Is the converse true ? Justify your answer. 5

(*ii*) Suppose (x_n) and (y_n) are two sequences such that :

$|x_n| \leq y_n$, for all $n \in N$, and $\lim (y_n) = 0$, then prove that $\lim(x_n) = 0$. 2½

4. (*a*) State Monotone Convergence Theorem and show that sequence (x_n) where $x_1 = 1$ and $x_{n+1} = \sqrt{2 + x_n}$, for all $n \geq 1$ is convergent. 5

(*b*) Using definition of Cauchy sequence, prove that sequence $\left(\frac{1}{n}\right)$ is a Cauchy sequence but the sequence $\left(n + \frac{(-1)^n}{n}\right)$ is not Cauchy. 5

(*c*) Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence. 5

5. (*a*) Suppose X = (x_n) is a bounded sequence of real numbers and let $x \in$ R have the property that every convergent subsequence of X converges to x. Then prove that the sequence X converges to x. 5

(*b*) (*i*) State Bolzano-Weierstrass Theorem for sequences. Give an example of an unbounded sequence that has a convergent subsequence. 2

(*ii*) Find lim sup and lim inf of sequences $\left(n^{1/n}\right)$ and $\left(5^{(-1)^n}\right)$. 3

(*c*) (*i*) Give an example of a sequence (x_n) which is not bounded below but lim sup $x_n = 50$. 2

(*ii*) For the sequence $a_n = \left[n + (-1)^n\right], n \in$ N, find its set of subsequential limits. 3

6. (*a*) State Cauchy criteria for convergence of a series of real numbers and hence show that the series $\sum \frac{1}{n}$ is not convergent. 5

(*b*) State Root Test for series of real numbers and show that the series $\sum \left(\sin \frac{n\pi}{3}\right)^n$ is convergent, but Root Test gives no information for the series $\sum \left(\sin \frac{n\pi}{2}\right)^n$. 5

(c) Examine the following series for convergence :

(i) $\sum \frac{(100)^n}{n!}$ 2½

(ii) $\sum \left[\sqrt{n+1} - \sqrt{n}\right]$ 2½

7. (a) Suppose $\Sigma\, a_n$ is a series where $a_n \geq 0\ \forall\ n$ and $|b_n| \leq a_n\ \forall\ n$. Then prove that $\Sigma\, a_n$ is convergent implies $\Sigma\, b_n$ is convergent. Is the series $\sum \left(\frac{2+\cos n}{3^n}\right)$ convergent ? Justify your answer. 5

(b) Give an example of a series which is convergent but not absolutely convergent. Justify your answer and show that $\sum \frac{(-1)^n \sin n\alpha}{n^3}$ is absolutely convergent. 5

(c) Examine the convergence of the following series :

(i) $\sum \frac{(-1)^n}{\sqrt{n}}$ 2½

(ii) $\sum \left[\frac{2}{(-1)^n - 3}\right]^n$ 2½