[This question paper contains 4 printed pages.]

1460-A *Your Roll No.* ....

B.A./B.Sc. (Hons.)/I **A**

MATHEMATICS – Unit I

(Vector, Calculus and Geometry)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **one** *question from each section. Marks are indicated against each question.*

SECTION I

1. (a) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is the time. Find the components of its velocity and acceleration at t = 1 in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. (3)

(b) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at (2, 1, 2) in the direction $2\hat{i} - 3\hat{j} + 6\hat{k}$. (3)

(c) If $\phi(x, y, z)$ is a scalar point function and $\overline{V}(x, y, z)$ is a vector point function, prove that $\text{curl}\left(\phi\vec{V}\right) = (\text{grad}\ \phi) \times \vec{V} + \phi\ \text{curl}\,\vec{V}$. (3½)

P.T.O.

2. (a) If $\vec{A} = x^2yz\hat{i} - 2xz^3\hat{j} + xz^2\hat{k}$ and $\vec{B} = 2z\hat{i} + y\hat{j} - x^2\hat{k}$, find $\frac{\partial^2}{\partial x \partial y}\left(\vec{A} \times \vec{B}\right)$ at $(1, 0, -2)$. (3)

(b) Find $\text{curl}\left[\vec{r}\, f(r)\right]$ where $f(r)$ is differentiable, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $|\vec{r}| = r$. (3)

(c) If $\vec{V_1}$ and $\vec{V_2}$ are two vector point functions, then prove that

$$\text{div}\left(\vec{V_1} \times \vec{V_2}\right) = \vec{V_2} \cdot \text{curl}\,\vec{V_1} - \vec{V_1} \cdot \text{curl}\,\vec{V_2}. \qquad (3\tfrac{1}{2})$$

SECTION II

3. (a) Find the co-axial system, one of whose members is $x^2 + y^2 + 2x + 3y - 7 = 0$ and a limiting point is $(1, -2)$. (4½)

(b) Show that the locus of points such that two of the normals drawn from them to the parabola $y^2 = 4ax$ coincide is

$$27ay^2 = 4(x - 2a)^3. \qquad (5)$$

4. (a) The tangent at a point P of an ellipse meets the auxilliary circle in two points which subtend a right angle at the centre. Show that the eccentricity of the ellipse is

$$\left(1 + \sin^2\phi\right)^{\frac{1}{2}}$$

where ϕ is the eccentric angle of the point P.

(4½)

(b) Show that the locus of the middle points of the chords of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

which subtend a right angle at the centre is

$$\left(\frac{x^2}{a^2} - \frac{y^2}{b^2}\right)\left(\frac{1}{a^2} - \frac{1}{b^2}\right) = \frac{x^2}{a^4} + \frac{y^2}{b^4}$$ (5)

SECTION III

5. Trace the conic :

$x^2 + 4y^2 - 4xy - 32x + 4y + 16 = 0.$ (9½)

6. Trace the conic :

$16x^2 - 24xy + 9y^2 + 77x - 64y + 95 = 0.$ (9½)

SECTION IV

7. (a) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point $(1, -2, 1)$ and intersects the sphere

$$x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$$

orthogonally. (4½)

(b) Find the equations of the lines of intersection of the plane $3x + 4y + z = 0$ and the cone $15x^2 - 32y^2 - 7z^2 = 0.$ (5)

8. (a) Three spheres of radii r_1, r_2 and r_3 have their centres at the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c) such that $r_1^2 + r_2^2 + r_3^2 = a^2 + b^2 + c^2$. A fourth sphere passes through the origin and the points A, B, C. Show that the radical centre of the four spheres lies on the plane

$$ax + by + cz = 0. \qquad (4½)$$

(b) Find the equation of the right circular cylinder which passes through the circle

$$x^2 + y^2 + z^2 = 9, \quad x - y + z = 3. \qquad (5)$$