**1460**

### B.Sc. (Hons.)/I    A

### MATHEMATICS—Paper I

### (Calculus)

### (Admissions of 2009 and onwards)

*Time : 3 Hours*                              *Maximum Marks* : 75

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*All* the Sections are compulsory.

Use of scientific calculator is allowed.

#### Section I

Attempt any *three* questions.

1.    Determine *a, b, c* such that the graph of $f(x) = ax^3 + bx^2 + c$ has an inflection point and slope 1 at $(-1, 2)$.                              4

2.    Show that, in general, the graph of the function

$$f(x) = \frac{ax^2 + bx + c}{rx^2 + sx + t}$$

will have $y = \dfrac{a}{r}$ as a horizontal asymptote and that when $br \neq as$ the graph will cross this asymptote at the point where

$x = \dfrac{at - cr}{br - as}.$                              4

3. Find A so that :

$$\lim_{x \to +\infty} \left( \frac{x + A}{x - 2A} \right)^x = 5.$$ 4

4. A manufacturer estimates that when $x$ units of a particular commodity are produced each month, the total cost (in dollars) will be

$$c(x) = \frac{1}{8}x^2 + 4x + 200$$

and all units can be sold at a price of $p(x) = 49 - x$ dollars per unit. Determine the price that corresponds to maximum profit. 4

## Section II

Attempt any *three* questions.

5. Show that if $\vec{u}, \vec{v}, \vec{w}$ are vectors in $R^3$ with $\vec{u} + \vec{v} + \vec{w} = 0$

then $\vec{u} \times \vec{v} = \vec{v} \times \vec{w} = \vec{w} \times \vec{u}$. 4½

6. The vector $\vec{r}(t) = (3 \cos t)\hat{i} + (3 \sin t)\hat{j} + t^2\hat{k}$ gives the position of a moving body at time $t$. Find the body's speed and direction when $t = 2$. At what time (if any) is the body's velocity and acceleration orthogonal ?                    4½

7. A boy standing at the edge of a cliff throws a ball upwards at an angle of 30° with an initial speed of 64 ft./sec. Suppose that when the ball leaves the boy's hand, it is 48 ft. above the ground at the base of the cliff. What are the time of flight of the ball and its range ?                    4½

8. Find an equation of the line which passes through the point Q(2, −1, 3) and is orthogonal to the plane $3x - 7y + 5z + 55 = 0$. Where does the line intersect the plane ?                    4½

## Section III

Attempt any *two* questions.

9. Identify and sketch the curve $x^2 + 9y^2 + 2x - 18y + 1 = 0$. 6

10. Find the distances from the pole to the vertices and find the equation of the ellipse in rectangular co-ordinates for

$$r = \frac{1}{2 - \cos \theta}.$$
6

11. Show that the graph of the equation $\sqrt{x} + \sqrt{y} = 1$ is a portion of the parabola. 6

12. Find an equation of the elliptical trace in the plane $x = 3$ of $4x^2 + 9y^2 + 18z^2 = 72$. Also find the length of the major and minor axis, and the coordinates of the foci of the ellipse. 6

## Section IV

.Attempt any *three* questions.

13. When two resistances $R_1$ and $R_2$ are connected in parallel, the total resistance R satisfies

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

If $R_1$ is measured as 300 ohms with a maximum error of 2% and $R_2$ is measured as 500 ohms with a maximum error of 3%, what is the maximum percentage error in R ?    4½

14. A paint store carries two brands of latex paint. An analysis of sales figure indicates that the demand Q for the first brand is modeled by

$$Q(x, y) = 210 - 12x^2 + 18y \text{ gallons/month}$$

where $x$ and $y$ are the prices of the first and the second brand respectively. A separate study indicates that $t$ months from now, the first brand will cost $x = 4 + 0.18\ t$ dollars/gallon

and the second brand will cost $y = 5 + 0.3\sqrt{t}$ dollars/gallon.
At what rate will the demand Q be changing with respect
to time 9 months from now ?                                    4½

15. Let $f(x, y, z) = xyz$ and let $\overrightarrow{u}$ be a unit vector perpendicular

to both $\overrightarrow{v} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\overrightarrow{w} = 2\hat{i} + \hat{j} - \hat{k}$. Find

the directional derivative of $f$ at $P_0(1, -1, 2)$ in the direction

of $u$.                                                        4½

16. The temperature T at point $(x, y, z)$ in a region of space is

given by the formula

$$T = 100 - xy - yz - zx.$$

Find the lowest temperature on the plane $x + y + z = 10.$  4½

## Section V

Attempt any *three* questions.

17. Sketch the graph of $r^2 = 4 \cos 2\theta$ in polar co-ordinates. 4

18. Find the equations of the paraboloid $z = x^2 + y^2$ in :

(*i*)  Cylindrical co-ordinates;

(*ii*) Spherical co-ordinates.                                    4

19. Show that the curvature of the circular helix :

$$x = a \cos t, \; y = a \sin t, \; z = ct$$

where $a > 0$, is $\dfrac{a}{a^2 + c^2}$.                          4

20. Find all values of $x$ where the graph of $y = \dfrac{2x^3 - 3x + 4}{x^2}$

crosses its oblique asymptote.                                    4

## Section VI

Attempt any *two* questions.

21. Evaluate

$$\int_0^{\pi/3} \sin^4 3x \cos^3 3x \, dx$$                       6

22. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$, about the line $x = 3$. 6

23. Find the area of the surface generated by revolving the curve $y = x^3$, $0 \leq x \leq \dfrac{1}{2}$, about the X-axis. 6

24. Find the length of the curve :

$$y = \frac{4\sqrt{2}}{3} x^{3/2} - 1, \ 0 \leq x \leq 1.$$ 6