This question paper contains 4 printed pages]

Roll No. | | 4 | 1 | 0 | 1 | 4 | 3 | 9 | 0 | 5 | 2 |

S. No. of Question Paper : **2443**

Unique Paper Code : **2352401** **F-4**

Name of the Paper : **Linear Algebra**

Name of the Course : **B.A./B.Sc. (H) — Allied Course**

[For the Students of other than Mathematics]

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* questions by selecting any *two* parts from each question.

1. (*a*) A woman rowing on a wide river wants the resultant (net) velocity of her boat to be 8 km/hr westward. If the current is moving 2 km/hr northeastward, what velocity vector should she maintain ? 6

(*b*) Find the angle between the vectors $x = [8, -20, 4]$ and $y = [6, -15, 3]$. Hence or otherwise comment whether x and y are parallel or not. 6

(*c*) Using Newton's Second Law of Motion, find the resultant sum of forces on a 30 kg object in a three-dimensional coordinate system undergoing an acceleration of 6 m/sec^2 in the direction of the vector $[-2, 3, 1]$. 6

P.T.O.

2. (*a*) If A and B are row equivalent matrices, then what is the relationship between Row Space of A and Row Space of B ? Also, determine whether the vector [2, 2, –3] is in the row space of the matrix : 6.5

$$A = \begin{bmatrix} 4 & -1 & 2 \\ -2 & 3 & 5 \\ 6 & 1 & 9 \end{bmatrix}.$$

(*b*) Find all eigenvalues corresponding to the given matrix. Also, express each eigenspace as a set of linear combinations of fundamental eigenvectors. The given matrix is : 6.5

$$B = \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}.$$

(*c*) (*i*) Show that the set ϕ of real-valued functions, f, defined on the interval [0, 1] such that $f\left(\frac{1}{2}\right) = 1$, is not a vector space under the usual operations of function addition and scalar multiplication.

(*ii*) Define subspace of a vector space. Prove or disprove that the set of 2-vectors of the form $[a, 2a]$ is a subspace of $\mathbf{R}^2$ under the usual vector operations, where a represents an arbitrary real number. 2+4.5

3. (*a*) Use the Simplified Span Method to find a simplified general form for all the vectors in Span (S), where S = {[1, 1, 0], [2, –3, –5]} is a subset of $\mathbf{R}^3$. 6

(*b*) Find if the homogeneous system $Ax = 0$, where :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 3 \end{bmatrix},$$

has a non-trivial solution, using rank of A. 6

(*c*) Show that the set $B = \{[1, -2, 1], [5, -3, 0]\}$ is the maximal linearly independent subset of $S = \{[1, -2, 1], [3, 1, -2], [5, -3, 0], [5, 4, -5], [0, 0, 0]\}$. Calculate dim(Span(S)). Is Span(S) = $\mathbf{R}^3$? Why or why not ? 6

4. (*a*) State the Dimension Theorem. Verify it for the linear transformation $L : \mathbf{R}^3 \to \mathbf{R}^2$ given by : 6.5

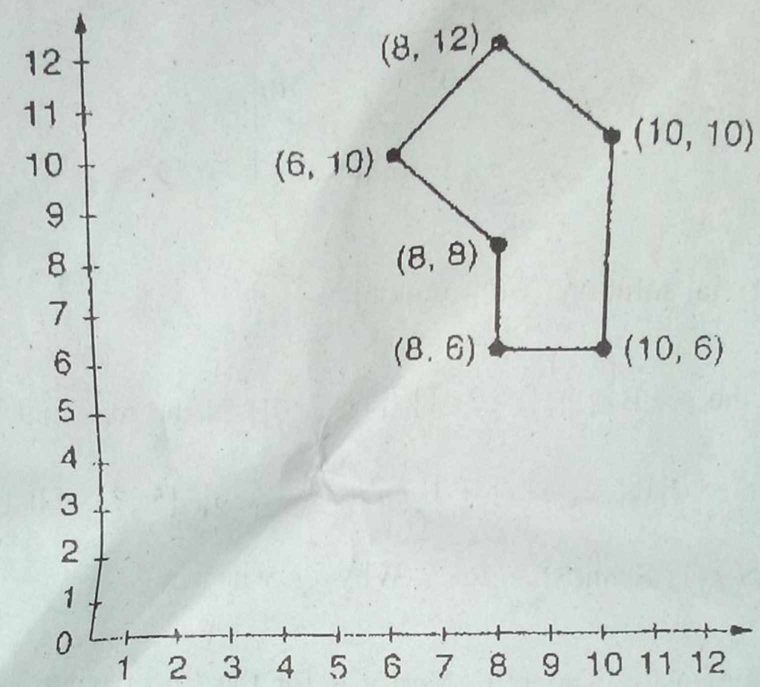
$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3 & 2 & 11 \\ 2 & 1 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

(*b*) Find the matrix for the linear transformation L with respect to the standard bases for P_3 and $\mathbf{R}^3$, where $L : P_3 \to \mathbf{R}^3$ is defined by :

$$L\left(a_3x^3 + a_2x^2 + a_1x + a_0\right) = \left[a_0 + a_1, 2a_2, a_3 - a_0\right].$$

Use this matrix to compute $L\left(5x^3 - x^2 + 3x + 2\right)$ by matrix multiplication. 6.5

(c) For the adjoining graphic, use ordinary coordinates in $\mathbf{R}^2$ to find the new vertices after performing a rotation about the point (12, 6) through an angle $\theta = 90°$. Then sketch the figure that would result from this movement. 6.5



5. (a) Let V be P_1 and let $S = \{v_1, v_2\}$ and $T = \{w_1, w_2\}$ be ordered bases for P_1, where $v_1 = t$, $v_2 = t - 3$, $w_1 = t - 1$, $w_2 = t + 1$. Compute the transition matrix $Q_{T \leftarrow S}$ from the S-basis to the T-basis. 6

(b) Define a linear operator. Show that the mapping $f : P_n \to P_n$ defined by :

$$f(p) = p + p'$$

is a linear operator on P_n. 6

(c) Is the linear transformation $L : \mathbf{R}^3 \to \mathbf{R}^3$ given by :

$$L([x, y, z]) = [2x, x + y + z, -y]$$

one-to-one ? Also, check whether it is onto or not. 6

6. (a) Show that the vector space $\mathbf{R}^4$ is isomorphic to the vector space P_3, using the transformation $L : \mathbf{R}^4 \to P_3$ given by $L([a, b, c, d]) = ax^3 + bx^2 + cx + d$. 6.5

(b) Verify that the set $B = \{[1, 0, -1], [-1, 4, -1], [2, 1, 2]\}$ is an orthogonal basis for $\mathbf{R}^3$. Obtain from B an orthonormal basis for $\mathbf{R}^3$. 6.5

(c) Find the orthogonal complement $W^{\perp}$ of the subspace $W = \{[a, b, 0] | a, b \in \mathbf{R}\}$ of $\mathbf{R}^3$. Verify that $\dim(W) + \dim(W^{\perp}) = \dim(\mathbf{R}^3)$. 6.5