[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 2370 **F-4** **Your Roll No.** 4101439032

Unique Paper Code : 2221402

Name of the Course : **B.Sc. (Hons.) Physics**

Name of the Paper : Mathematical Physics - III

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Atleast one question from each section.

SECTION A

1. (a) Express the complex number $2\sqrt{3} - 2i$ in polar form. (3)

   (b) Find the cube roots of 8. (3)

   (c) Derive the necessary and sufficient condition for the complex function to be analytic. (6)

   (d) Using the Cauchy-Riemann equations, show that $f(Z) = Z^3$ is analytic in the entire Z-plane (3)

2. (a) Evaluate $\int \frac{e^z}{1+Z^2} dZ$ over the circle $|z| = 2$ (5)

   (b) If f(Z) is analytic inside and on the boundary C of a simply connected region R, prove that

$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(Z)}{Z-a} dZ$$ (5)

(c) Find the residue of $f(Z)=\dfrac{Ze^{z}}{(Z-a)^{3}}$ at its poles. (5)

3. (a) Expand $f(Z)=\dfrac{1}{(Z-2)^{2}}$ in aLaurent series valid for $|Z|<2$ (5)

(b) Expand $f(Z)$ = sin$Z$ in a Taylor series about $Z=\dfrac{\pi}{4}$ and (5)

(c) Locate and name all the singularities of the function

$$f(Z)=\frac{Z^{8}+Z^{4}+2}{(Z-1)^{3}+(3Z+2)^{3}}$$ (5)

4. Using contour integration, evaluate any two of the following.

(a) $\displaystyle\int_{0}^{\infty}\frac{dx}{x^{4}+1}$

(b) $\displaystyle\int_{0}^{\pi}\frac{\sin 3\theta}{5-3\cos\theta}\,d\theta$

(c) $\displaystyle\int_{0}^{\infty}\frac{\sin^{2}x}{x^{2}}\,dx$ (7½, 7½)

SECTION B

5. (a) Find the Fourier Transform of $f(x)=\begin{cases}1-x, & |x|<1\\ 0, & |x|>1\end{cases}$ (7½)

(b) Verify the convolution theorem of Fourier transform for the function

$$f(x) = g(x) = \begin{cases} 1, & |x| < 1 \\ 0 & |x| > 1 \end{cases}$$ (7½)

6. Using the Fourier Transform solve

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} \qquad x > 0, \quad t > 0$$

subject to the conditions

(i) $u(0, t) = 0$

(ii) $u(x, 0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$

(iii) $u(x, t)$ *is bounded* (12)

(b) If $g(\alpha)$ is the Fourier Transform of $f(x)$ them the Fourier Transform of $f(ax)$ is $\frac{1}{a} g(\frac{\alpha}{a})$ (3)

SECTION C

7. (a) If L{F(t)}=f(s) than prove that $L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(s)\, ds$ (5)

(b) Use convolution theorem to find $L^{-1}\left(\frac{1}{s^2(s^2+a^2)}\right)$ (5)

(c) Prove that $L\{\delta(t)\} = 1$ *where* $\delta(t)$ is *Dirac delta function* (5)

8. (a) Using Laplace transform solve $\frac{dx}{dt}+y=0$ *and* $\frac{dy}{dt}-x=0$ (8)

where $x(0) = 1$ *and* $y(0) = 0$

(b) Find the Laplace transform of the function $F(t)=\begin{cases} 1, & 0\leq t<1 \\ t, & 1\leq t<2 \\ t^2, & 2\leq t<=\infty \end{cases}$ (7)