This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1582**

Unique Paper Code : **222202** **C**

Name of the Paper : **Oscillations and Waves (PHHT-204)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all including Q. No. **1** which is compulsory.

1. Do any *five* of the following : 5×3=15

(*a*) A uniform solid sphere of mass M, radius R and centre at C executes SHM about its tangent [as shown in Fig. (1)]. Find the time period of oscillation.

[Given : moment of inertia of sphere about tangent is 7 $MR^2/5$]

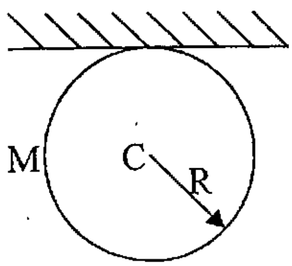

Fig. 1

(*b*) A string of length 3L and negligible mass is attached to two fixed ends. The tension in the string is T. A particle of mass m is attached at a distance L from one end of the string [as shown in Fig. 2]. Find the time period of the small transverse oscillations of mass m.

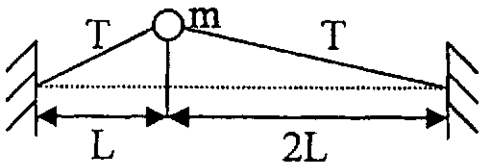

Fig. 2

(*c*) For a wave in medium, the angular frequency ω and wave vector $\vec{k}$ are related by :

$$\omega^2 = c^2k^2\left(1 + \alpha k^2\right),$$

where c and α are constants. Prove that the product of group velocity and phase velocity is given by :

$$v_g \cdot v_p = c^2\left(1 + 2\alpha k^2\right).$$

(*d*) A uniform string of length L and linear density μ is stretched with tension T between the fixed ends at $x = 0$ and $x = L$. If it is plucked at $x = \frac{L}{4}$, through a transverse height H. write expression for initial displacement, $y_0(x)$.

(*e*) Calculate the velocity of sound in :

(*i*) water and

(*ii*) steel.

Given density of water = 1000 kg/m^3, density of steel = 7800 kg/m^3, bulk modulus of water = 0.20×10^{10} N/m^2 and Young's modulus of steel = 20×10^{10} N/m^2.

(*f*) A spring of length L and force constant k, is cut into 2 pieces of lengths L_1 and L_2, such that $L_1 = nL_2$, where n is an integer. If k_1 and k_2 are force constants of the 2 pieces, respectively, then show that $k_2 = nk_1$.

(*g*) Find the fundamental, first overtone and second overtone frequencies of an open organ pipe of length 20 cm.

[Speed of sound in air is 340 m/s]

(*h*) What do you understand by Lissajous figure ? Draw the Lissajous figure (with direction) if the two perpendicular SHMs $x = 3\cos(\omega t + \alpha)$ and $y = 2\cos(\omega t + \beta)$ such that $\alpha - \beta = \frac{\pi}{2}$ act on a particle simultaneously.

2. (*a*) What do you understand by Centre of Percussion ? Find its position if a thin uniform rod of mass M and length L is fixed at its one end. 2,4

(*b*) A particle is subjected to two perpendicular SHMs simultaneously :

$$x = A_1 \cos(2\omega t + \alpha),\ y = A_2 \cos \omega t.$$

Obtain Lissajous Figures (analytically or graphically) if $\alpha = \frac{\pi}{2}$ and π. 6

(*c*) What do you understand by Q, the quality factor ? What is its value for an ideal oscillator ? 3

3. (*a*) Consider a mass M attached with two identical massless springs having spring constant k, relaxed length a_0 and equilibrium length a. If $\frac{a_0}{a}$ can be neglected (slinky approximation), show that longitudinal oscillations and transverse oscillations have the same frequency. 6

(*b*) Two equal masses *m* are connected with two identical massless springs of spring constant *k* [as shown in Fig. 3]. Show that the angular frequency of the two normal modes of vertical oscillation are given by :

$$\omega = \left(3 \pm \sqrt{5}\right)\frac{k}{2m}.$$

Also show that in the slower mode the ratio of the amplitude of mass 1 to that of mass 2 is $\frac{1}{2}\left(\sqrt{5} - 1\right)$ while in faster mode this ratio is $\frac{1}{2}\left(\sqrt{5} + 1\right)$. 5,4

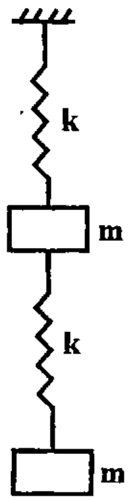

Fig. 3

4. (*a*) A particle of mass *m* is executing oscillatory damped harmonic motion. If *k* is the force constant and *b* is the damping coefficient, then prove that time-averaged total energy of the oscillator is given by :

$$\langle E(t)\rangle = \frac{1}{2}kA^2e^{-bt/m};$$

A is amplitude. 10

(*b*) Two points on the surface of the earth are joined by a straight smooth tunnel, not passing through the centre of the earth. A particle is dropped inside the tunnel. Prove that this mass executes SHM and hence find its time period. 2,3

5. (*a*) A uniform string of length L and linear density μ is stretched with tension T between the fixed ends at $x = 0$ and $x = L$. The general expression for transverse displacement $y(x, t)$ of the string is given by :

$$y(x, t) = \sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\left(A_n \cos\frac{n\pi vt}{L} + B_n \sin\frac{n\pi vt}{L}\right)$$

where, A_n and B_n are arbitrary constants and $v = \sqrt{\frac{T}{\mu}}$.

Prove that the total energy of the vibrating string is given by :

$$E_{total} = \frac{\mu\pi^2 v^2}{4L} \sum_{n=1}^{\infty} n^2\left(A_n^2 + B_n^2\right).$$ 9

(*b*) What do you understand by plucked string ? Prove that the energy of the string plucked at $x = \frac{L}{3}$, through a transverse height h is given by :

$$E_{total} = \frac{\mu}{L}\left(\frac{9hv}{2\pi}\right)^2 \sum_{n=1}^{\infty} \frac{1}{n^2}\sin^2\frac{n\pi}{3}.$$ 1,5

6. (*a*) Four equal masses m are equally spaced along a string of length 5L [as shown in Fig. 4]. The tension in the string is T. Show that the four normal mode frequencies and the corresponding amplitudes of normal mode displacements are :

$$\omega_1 = 2\omega_0 \sin\frac{\pi}{10}; \qquad A_1 = \alpha C,\ A_2 = \beta C,\ A_3 = \beta C,\ A_4 = \alpha C$$

$$\omega_2 = 2\omega_0 \sin\frac{\pi}{5}; \qquad A_1 = \beta C,\ A_2 = \alpha C,\ A_3 = -\alpha C,\ A_4 = -\beta C$$

$$\omega_3 = 2\omega_0 \sin\frac{3\pi}{10}; \qquad A_1 = \beta C,\ A_2 = -\alpha C,\ A_3 = -\alpha C,\ A_4 = \beta C$$

$$\omega_4 = 2\omega_0 \sin\frac{2\pi}{5}; \qquad A_1 = \alpha C,\ A_2 = -\beta C,\ A_3 = \beta C,\ A_4 = -\alpha C$$

Hence, draw the normal modes of transverse oscillations. 10,2

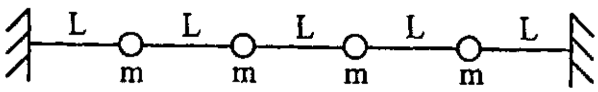

Fig. 4

$$\left[\text{Given} : \omega_n = 2\omega_0 \sin\frac{n\pi}{2(N+1)},\ A_p = C\sin\frac{pn\pi}{N+1},\ \alpha = \sin 36°,\ \beta = \sin 72°\right]$$

(*b*) The equation of a wave is given by :

$$y = A\sin(\omega t - k_1 x - k_2 y - k_3 z).$$

Calculate the wavelength and magnitude of velocity of the wave. 3

7. (*a*) Define ripple and gravity waves in terms of critical wavelength. Prove that the expression of the magnitude of the velocity of the waves formed on the surface of a liquid (density, ρ) under the combined action of gravity and surface tension T is given by :

$$v = \sqrt{\frac{\lambda g}{2\pi} + \frac{2\pi T}{\rho\lambda}},$$

λ = wavelength of the wave. 2,8

(*b*) What are group and phase velocities ? Prove that relation between them is given by :

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}.$$ 2,3

8. (*a*) Standing waves are formed in a pipe of length L with :

(*i*) both ends open, and

(*ii*) one end open and the other closed.

The particle displacement is given by :

$$y(x, t) = (A \sin kx + B \cos kx) \cos \omega t, \text{ (where, } k = 2\pi/\lambda)$$

and the boundary conditions are shown in Fig. 5. Prove that :

(*i*) $y(x, t) = B \cos kx \cos \omega t$, with $\lambda = 2L/n$ and

(*ii*) $y(x, t) = B \cos kx \cos \omega t$, with $\lambda = 4L/(2n + 1)$.

Sketch the first three harmonics for each case. 9

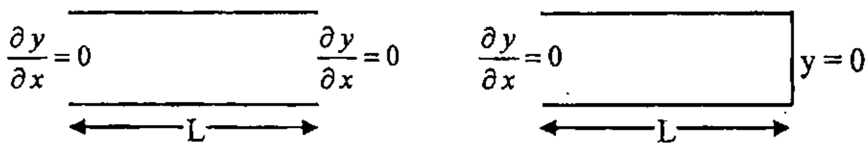

Fig. 5

(*b*) Prove that magnitude of velocity of transverse waves in a stretched string is given by :

$$v = \sqrt{\frac{T}{\mu}},$$

where, T is the tension in the string and μ is its linear mass density. 6