Department of Physics and Astrophysics
University of Delhi

PHY 401 : CLASSICAL MECHANICS M.Sc. / I Sem. November 2015

Time : Three hours *Maximum Marks* : 70

(Answer any five questions.)

1. Consider a particle of mass m executing motion in three dimensions under a central force potential given by

$$V(r) = Br^a,$$

where a and B are constants and r denotes the radial distance from the origin of the coordinate system. The angular momentum of the particle is J and the total energy is E.

(a) Calculate the minimum permissible value of energy E for a given J.

(b) Derive the expression for the radius of a stable circular orbit.

(c) Calculate the frequency of small radial oscillations about the stable circular orbit.

(d) For $a = 2$ and $B > 0$, calculate the turning points of the orbit for a given energy E.

[4+6+2+2]

2. Two particles of masses m_1 and m_2 move along the x-axis and their potential and kinetic energies are given, respectively, by

$$V = \frac{1}{2}k_1(x_1 - a)^2 + \frac{1}{2}k_2(x_2 + a)^2 + 2\alpha(x_1 - x_2)^2,$$

and

$$T = \frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2.$$

Here k_1, k_2, a and α are constants.

(a) Using the Lagrangian formalism, obtain the frequencies of oscillations ω_{10} and ω_{20} for m_1 and m_2 for the special case when $\alpha = 0$.

(b) Using the theory of small oscillations, obtain the frequencies ω_1 and ω_2 of the normal modes for the general case $\alpha \neq 0$ in terms ω_{10} and ω_{20}.

(c) Obtain the normal mode coordinates for the case of $\alpha \neq 0$.

[2+6+6]

3. The Lagrangian of a particle in one dimension is given by $L = \frac{1}{2}x^{-a}\dot{x}^2 - \lambda \mid x \mid^b$.

(a) Set up the equations of motion for this system.

(b) Calculate the Hamiltonian of the system.

(c) If the energy of the particle is E, what is the condition for oscillatory motion?

(d) If the amplitude of oscillation is increased by a factor α, how does the time period of oscillation change?

[2+3+3+6]

4. A particle in 3 dimensions moves in a potential given by,

$$V(r) = 0 \quad for \quad r \geq a \quad and \quad V(r) = -V_0 \quad for \quad r < a,$$

where, a and V_0 are positive constants.

(a) Calculate the scattering angle as a function of the impact parameter.

(b) Calculate the differential cross-section for scattering.

[8+6]

5. A set of phase-space coordinates (q, p) are transformed to another set (Q, P), according to the transformation defined by

$$Q = \ln[1 + \sqrt{q}\ \cos(p)] \quad and \quad P = 2\ \sqrt{q}\ \sin(p)\ [1 + \sqrt{q}\cos(p)].$$

(a) Show that the coordinates (Q, P) are canonical variables if the coordinates (q, p) are canonical.

(b) Calculate the generating function of third kind $F_3(Q, p)$ that generates the transformation between (q, p) and (Q, P).

[6+8]

6. Consider a particle of mass m moving along the x-axis in a potential $V(x) = \lambda \mid x \mid$, where λ is a positive constant.

(a) Construct the Lagrangian for the system.

(b) Find the corresponding Hamiltonian.

(c) If the particle starts from rest at $x = x_0 > 0$, sketch its phase-space trajectory.

(d) If the parameter λ is NOT a constant but is a slowly varying function of time, how does the amplitude of oscillations vary with λ?

[2+2+5+5]